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Computing Maximum Reachability Probabilities in Markovian Timed Automata

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Abstract. We propose a novel stochastic extension of timed automata, i.e. *Markovian Timed Automata* (MTA). We study the problem of optimizing the reachability probabilities in this model. Two variants are considered, namely, the time-bounded and unbounded reachability. In each case, we propose Bellman equations to characterize the probability. For the former, we provide two approaches to solve the Bellman equations, namely, the discretization and a reduction to Hamilton-Jacobi-Bellman equations. For the latter, we show that in the single-clock case, the problem can be reduced to solving a system of linear equations, whose coefficients are the time-bounded reachability probabilities in CTMDPs.

1 Introduction

This paper introduces Markovian timed automata (MTA, for short), an extension of timed automata [AD94] with exponential location residence times. An MTA has as semantics a *continuous-space* Markov decision process (MDP). MTA are rather expressive. Zero-clock MTA correspond to a subclass of continuous-time Markov decision processes (CTMDP, [BHKH05,BFK$^+$09]), whereas probabilistic timed automata (PTAs) [KNSS02] are obtained by basically ignoring the exit rates in any location. In earlier work [CHKM09a], we have used *deterministic* MTA as specification formalism for linear real-time properties over stochastic processes.

This paper focuses on determining maximal reachability probabilities in MTA. Two variants are considered: what is the maximal likelihood to hit a set of goal locations within a given deadline, and how is this probability determined in absence of such deadline? To solve these issues, we adopt the standard region construction [AD94] to MTA, and show that region graphs are in fact a decision variant of piecewise-deterministic Markov processes (PDPs) [Dav84], a well-studied class of stochastic processes in e.g., stochastic control theory and financial mathematics. In this paper, this variant is termed as *piecewise-deterministic Markov decision processes* (PDDPs, for short). We characterize maximal time-bounded reachability probabilities in PDDPs by a variant of the Bellman equation. This provides the basis for two approaches to compute such probabilities. The first approach uses discretization, and shows that max-reachability probabilities in a PDDP can be reduced to maximal reachability probabilities in an MDP, for which various efficient algorithms, such as value iteration [Ber95] exist. We show that the accuracy of our result is $(1 - e^{-\lambda h})\cdot(1 - e^{-\lambda T})$ where h is the discretization step, T is the deadline, and λ is the maximal rate of an exponential distribution in the MTA. The second approach is based on partial differential equations (PDEs), in particular Hamilton-Jacobi-Bellman equations. Finally, we provide a Bellman equation for time-unbounded reachability probabilities in PDDPs, and show that for one-clock MTA solving a linear equation whose coefficients are reachability probabilities in (locally uniform) CTMDPs [NSK09,BFK$^+$09]. In this paper, we focus on the *maximum* probabilities. However, all the results can be adapted to the *minimum* ones in a straightforward way.

Some related works are in order: [BBB$^+$07,BBB$^+$08,BBBM08] provides a quantitative interpretation to timed automata where delays and discrete choices are interpreted probabilistically. In this approach, delays of unbounded clocks are governed by exponential distributions like in CTMCs. Decidability results have been obtained for almost-sure properties [BBB+08] and quantitative verification [BBBM08] for (a subclass of) single-clock timed automata. This probabilistic semantics roughly corresponds to *deterministic* MTA, as considered in [CHKM09a]. Moreover, [BF09] considered stochastic timed games. Time-unbounded reachability problems were addressed for these models. It was shown that the problem is undecidable in general, and becomes decidable if restricting to single-clock $1\frac{1}{2}$-player games and qualitative case. MTA are essentially $1\frac{1}{2}$-player stochastic timed games. However, we are mainly dealing with *approximated quantitative* analysis of both time-bounded and unbounded cases.

2 Markov processes with decision

Given a set H, let $\Pr : \mathcal{F}(H) \to [0,1]$ be a probability measure on the measurable space $(H, \mathcal{F}(H))$, where $\mathcal{F}(H)$ is a σ-algebra over H. Let $Distr(H)$ denote the set of probability measure on this measurable space.

Definition 1 (MDP). *A* Markov decision process *is a tuple* $\mathcal{D} = (Act, S, s_0, \mathcal{P})$ *where* Act *is a* finite *set of* actions*;* S *is a set of* states*;* s_0 *is the* initial state*;* $\mathcal{P} : S \times Act \times \mathcal{F}(S) \to [0,1]$ *is the* transition probability function*, where* $\mathcal{P}(\cdot, \cdot, A)$ *is measurable for any* $A \in \mathcal{F}(S)$*.*

$\mathcal{P}(s, \alpha, A)$ is the one-step transition probability from state $s \in S$ to the set of states $A \in \mathcal{F}(S)$ by taking action $\alpha \in Act$. We assume w.l.o.g. that there is no internal nondeterminism, i.e., the actions enabled at each state are pairwise different.

Piecewise-deterministic Markov decision processes. The model *piecewise-deterministic Markov processes* (PDPs) [Dav84] constitute a general model for virtually any stochastic system without diffusions [Dav93] and has been applied to a variety of problems in engineering, operations research, management science, and economics. Powerful analysis and control techniques for PDPs have been developed [LL85,LY91,CD88].

A *piecewise-deterministic (Markov) decision process* (PDDP) is a PDP with decisions where the exit rate function, the flow function and the transition probability function might depend on a set of parameters (actions). Generally a PDDP does *not* define any stochastic process, since it is inherently *nondeterministic*. By resolving the nondeterminism (i.e., specifying all actions in PDDP) a PDP is derived. In this paper we consider a simple versions of PDDPs where only the transition probability function depends on a given action.

Let us first introduce some notions. Let $\mathcal{X}{=}\{x_1, ..., x_n\}$ be a set of variables in $\mathbb{R}$. An $\mathcal{X}$-valuation is a function $\eta{:}\mathcal{X}{\to}\mathbb{R}$ assigning to each variable x a value $\eta(x)$. Let $\mathcal{V}(\mathcal{X})$ denote the set of all valuations over $\mathcal{X}$. A *constraint* over $\mathcal{X}$, denoted by g, is a subset of $\mathbb{R}^n$. Let $\mathcal{B}(\mathcal{X})$ denote the set of constraints over $\mathcal{X}$. An $\mathcal{X}$-valuation η *satisfies* constraint g, denoted $\eta \models g$, if $(\eta(x_1), ..., \eta(x_n)) \in g$. For $g{\in}\mathcal{B}(\mathcal{X})$, a constraint over $\mathcal{X}{=}\{x_1, ..., x_n\}$, let $\overline{g}$ be the closure of g, $\mathring{g}$ the interior of g, and $\partial g = \overline{g} \setminus \mathring{g}$ the boundary of g, e.g., for $g{=}x_1^2{-}2x_2{\leqslant}1.5 \wedge x_3{>}2$, we have $\mathring{g}{=}x_1^2 - 2x_2{<}1.5 \wedge x_3{>}2$, $\overline{g}{=}x_1^2{-}2x_2{\leqslant}1.5 \wedge x_3{\geqslant}2$, and ∂g equals $x_1^2{-}2x_2{=}1.5 \wedge x_3{=}2$.

A PDDP is a hybrid stochastic decision process involving discrete control (i.e., locations) and continuous variables. To each control location z of a PDDP, an *invariant* $Inv(z)$ is associated, a constraint over $\mathcal{X}$ which constrains the variable values in z. The state of a PDDP is a pair (z, η) with control location z and η a variable valuation. For the set Z of locations, let $\mathbb{S} = \{\, (z, \eta) \mid z \in Z, \eta \models Inv(z) \,\}$ be the state space of the PDDP. The notions of closure, interior and boundary can be lifted to $\mathbb{S}$ in a straightforward manner, e.g., $\partial\mathbb{S} = \bigcup_{z \in Z}\{z\} \times \partial Inv(z)$ is the boundary of $\mathbb{S}$; $\mathring{\mathbb{S}}$ and $\overline{\mathbb{S}}$ are defined in a similar way.

Definition 2 (PDDP [Dav93]). *A* PDDP *is a tuple* $\mathcal{Z} = (Act, Z, \mathcal{X}, Inv, \phi, \Lambda, \mu)$ *where* Act *is a finite set of* actions*;* Z *is a finite set of* locations*;* $\mathcal{X}$ *is a finite set of* variables*;* $Inv : Z \to \mathcal{B}(\mathcal{X})$ *is an* invariant function*;* $\phi : Z \times \mathcal{V}(\mathcal{X}) \times \mathbb{R} \to \mathcal{V}(\mathcal{X})$

is a flow function[1]*;* $\Lambda : \mathbb{S} \to \mathbb{R}_{\geqslant 0}$ *is an* exit rate function *satisfying for any* $\xi \in \mathbb{S}$*:* $\exists \epsilon(\xi) > 0.$ *function* $t \mapsto \Lambda(\xi \oplus t)$ *is integrable on* $[0, \epsilon(\xi))$*, where* $(z, \eta) \oplus t = \big(z, \phi(z, \eta, t)\big)$ *and* $\mu : (\mathring{\mathbb{S}} \cup \partial\mathbb{S}) \times Act \times \mathcal{F}(\mathbb{S}) \to [0, 1]$ *is the* transition probability function *satisfying*[2]*:* $\mu(\xi, \alpha, \{\xi\}) = 0$*, where* $\mathcal{F}(\mathbb{S})$ *is a* σ*-algebra generated by the union* $\bigcup_{z \in Z} \{z\} \times A_z$ *with* $A_z \subseteq \mathcal{F}(Inv(z))$*.*

Let us explain the behavior of a PDDP. A PDDP can reside in a location z as long as $Inv(z)$ holds. On entering state $\xi = (z, \eta)$, the PDDP can either *delay* or take a *Markovian jump*. By delaying, the next state $\xi' = \xi \oplus t$, i.e., the PDDP remains in location z while all its continuous variables are updated according to $\phi(z, \eta, t)$. The flow function ϕ defines the time-dependent behavior in a single location, in particular, how the variable valuations change when time elapses. State $\xi \oplus t$ is the timed successor of state ξ (on the same location) given that t time units have passed. In case of a Markovian jump, the next state $\xi'' = (z'', \eta'') \in \mathbb{S}$ is reached with probability $\mu(\xi, \alpha, \{\xi''\})$ by taking action α. The residence time of a state is exponentially distributed; this is defined by the function Λ. A third possibility for a PDDP to evolve is by taking *forced transitions*. When the variable valuation η satisfies the boundary, i.e., $\eta \models \partial Inv(z)$, the PDDP is forced to take a boundary jump, i.e., it has to leave location z. With probability $\mu(\xi, \alpha, \{\xi''\})$ it then moves to state ξ'' for arbitrary action $\alpha \in Act$.

The PDP is piecewise-deterministic because in each location (one piece) the behavior is deterministically determined by ϕ. The decision process is Markovian as the current state contains all the information to determine the future progress of the process.

3 Markovian timed automata

We use a special case of *nonnegative* variables, called *clocks*. We write $\vec{0}$ for the valuation that assigns 0 to all clocks. For a subset $X \subseteq \mathcal{X}$, the reset of X, denoted $\eta[X := 0]$, is the valuation η' such that $\forall x \in X.\ \eta'(x) = 0$ and $\forall x \notin X.\ \eta'(x) = \eta(x)$. For $\delta \in \mathbb{R}_{\geqslant 0}$, $\eta + \delta$ is the valuation η'' such that $\forall x \in \mathcal{X}.\ \eta''(x) = \eta(x) + \delta$, which implies that all clocks proceed at the same speed, or equivalently, $\forall x_i \in \mathcal{X}.\ \dot{x_i} = 1$. A *clock constraint* on $\mathcal{X}$ is an expression of the form $x \bowtie c$, or the conjunction of clock constraints, where $x \in \mathcal{X}$, $\bowtie \in \{<, \leqslant, >, \geqslant\}$ and $c \in \mathbb{N}$.

Definition 3 (MTA). *An* MTA *is a tuple* $\mathcal{M} = (Act, \mathcal{X}, Loc, \ell_0, E, \rightsquigarrow)$*, where* Act *is a finite set of* actions*;* $\mathcal{X}$ *is a finite set of* clocks*;* Loc *is a finite set of* locations*;* $\ell_0 \in Loc$ *is the* initial location*;* $E : Loc \to \mathbb{R}_{\geqslant 0}$ *is the* exit rate function*; and* $\rightsquigarrow \subseteq Loc \times Act \times \mathcal{B}(\mathcal{X}) \times Distr(2^{\mathcal{X}} \times Loc)$ *is the* edge relation*.*

Let $\mathcal{I}(\ell, \eta) \in Act$ be the set of actions enabled in location $\ell \in Loc$ under clock valuation $\eta \in \mathcal{V}(\mathcal{X})$. For simplicity we abbreviate $(\ell, \alpha, g, \zeta) \in \rightsquigarrow$ by $\ell \overset{\alpha, g}{\rightsquigarrow} \zeta$, where ζ is a probability distribution over $2^{\mathcal{X}} \times Loc$. Compared to the DMTA in [CHKM09a], where the edge relation is defined as $\rightsquigarrow \subseteq Loc \times \mathcal{B}\ (\mathcal{X}) \times 2^{\mathcal{X}} \times Distr(Loc)$, the MTA model allows each set of transitions to reset their clocks differently. This has also been used in *probabilistic timed automata* (PTA, [KNSS02]). In this sense, our model can be considered as a continuous-time extension of PTAs. The locally uniform [BHKH05] *continuous-time* MDP*s* (CTMDPs) [NSK09] with *finite* state space are zero-clock MTAs (i.e., $\mathcal{X} = \varnothing$).

Example 1. An example MTA is shown in Fig. 1, where there are 4 locations with ℓ_0 the initial location. Note that the sub-probability distribution can be fixed by adding a *trap* location. The resets are over distributions. In ℓ_0, there is a nondeterministic choice between α_1 and α_2. It is similar for ℓ_2 with γ_1 and γ_2. ℓ_3 is the *goal* location, which will be used later.denoted by a double cycle.

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[1] The flow function is the solution of a system of ODEs with a Lipschitz continuous vector field.

[2] $\mu(\xi, \alpha, A)$ is a shorthand for $(\mu(\xi, \alpha))(A)$.

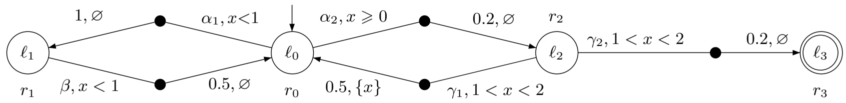

Fig. 1. An example MTA

Definition 4. *Finite paths in* MTA $\mathcal{M}$ *are of the form* $\ell_0 \xrightarrow{\alpha_0,t_0} \ell_1 \xrightarrow{\alpha_1,t_1} \cdots \ell_n$, *where for each edge* $\ell_i \overset{\alpha_i,g_i}{\rightsquigarrow} \zeta_i$ *of* $\mathcal{M}$ *with* $\zeta_i(X_i, \ell_{i+1}) > 0$ ($\ell_i \in Loc$, $\alpha_i \in Act$, $t_i \in \mathbb{R}_{\geq 0}$, $X_i \subseteq \mathcal{X}$ *and* $0 \leqslant i < n$), *we have that* η_i *is a valid clock valuation on* entering *location* ℓ_i *satisfying* $\eta_0 = \vec{0}$, $(\eta_i + t_i) \models g_i$, *and* $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$. *Let* $Paths^{\mathcal{M}}$ *(resp.* $Paths^{\mathcal{M}}(\ell)$*) denote the set of finite paths (resp. starting in* ℓ*) in* $\mathcal{M}$. *For* $\rho \in Paths^{\mathcal{M}}$, *let* $\rho[n] := \ell_n$ *be the* n*-th location of* ρ *and* $\rho\langle n\rangle := t_n$ *be the time spent in* ℓ_n.

Semantics. The semantics of an MTA is given as an MDP. A *state* of an MTA is of the form (ℓ, η) where $\ell \in Loc$ and $\eta \in \mathcal{V}(\mathcal{X})$ is a clock valuation.

Definition 5. *Let* $\mathcal{M} = (Act, \mathcal{X}, Loc, \ell_0, E, \rightsquigarrow)$ *be an* MTA. *The* MDP *associated with* $\mathcal{M}$ *is* $\mathcal{D}(\mathcal{M}) = (Act, S, s_0, \mathcal{P})$ *where* $S = Loc \times \mathcal{V}(\mathcal{X})$, $s_0 = (\ell_0, \vec{0})$, *and for each edge* $\ell \overset{\alpha,g}{\rightsquigarrow} \zeta$ *in* $\mathcal{M}$ *with* $\zeta(X, \ell') = p > 0$ *and any* $\eta \models g$, *we have:*

$$\mathcal{P}((\ell,\eta),\alpha,A) = \int_{\mathbb{R}_{\geqslant 0}} E(\ell)e^{-E(\ell)\tau} \cdot \mathbf{1}_g(\eta+\tau) \cdot p \, d\tau, \tag{1}$$

where $A = \{(\ell', \eta') \mid \exists \tau \in \mathbb{R}_{\geqslant 0}.\ \eta' = (\eta+\tau)[X:=0]$ *and* $\eta + \tau \models g\}$ *and* $\mathbf{1}_g(\cdot)$ *is the characteristic function, i.e.,* $\mathbf{1}_g(\eta+\tau) = 1$ *if* $\eta + \tau \models g$; 0, *otherwise.*

Intuitively, an MTA behaves as follows. Given an edge $\ell \overset{\alpha,g}{\rightsquigarrow} \zeta$ such that $\zeta(X, \ell') = p > 0$, i.e., there is a transition from ℓ to ℓ' under action α, if more than one action is enabled in state (ℓ, η) then one of them is chosen nondeterministically. Given that action α is chosen, consider an interval $I = [t_1, t_2] \subseteq \mathbb{R}_{\geqslant 0}$ such that for each $t \in I$, $\eta + t \models g$. The probability of taking the α-transition from state (ℓ, η) to the set of states $A = \{(\ell', (\eta+t)[X := 0]) \mid t \in I\}$ is $p \cdot (e^{-E(\ell)\cdot t_1} - e^{-E(\ell)\cdot t_2})$.

We now consider the probability of a set of paths. Given an MTA $\mathcal{M}$, $C(\ell_0, \alpha_0, I_0, ..., \alpha_{n-1}, I_{n-1}, \ell_n)$ (C for short) is the cylinder set where $(\ell_0, ..., \ell_n) \in Loc^{n+1}$ and $I_i \subseteq \mathbb{R}_{\geqslant 0}$. The cylinder set denotes a set of infinite paths ρ in $\mathcal{M}$ such that $\rho[i] = \ell_i$ and $\rho\langle i\rangle \in I_i$. Let $\Pr^{\mathcal{M}}_{\eta_0}(C)$ denote the probability of C such that the initial clock valuation in location ℓ_0 is η_0. Let $\Pr^{\mathcal{M}}_{\eta_0}(C) := \mathbb{P}^{\mathcal{M}}_0(\eta_0)$, where $\mathbb{P}^{\mathcal{M}}_i(\eta)$ is defined as follows:

$$\mathbb{P}^{\mathcal{M}}_i(\eta) = \begin{cases} 1 & \text{if } i = n \\ \displaystyle\int_{I_i} \underbrace{E(\ell_i) \cdot e^{-E(\ell_i)\tau} \cdot \mathbf{1}_{g_i}(\eta+\tau) \cdot p_i}_{(\star)} \cdot \underbrace{\mathbb{P}^{\mathcal{M}}_{i+1}(\eta')}_{(\star\star)} \, d\tau & \text{if } 0 \leqslant i < n, \end{cases} \tag{2}$$

where $\eta' := (\eta+\tau)[X_i := 0]$. Intuitively, $\mathbb{P}^{\mathcal{M}}_i(\eta_i)$ is the probability of the cylinder set C starting from ℓ_i and η_i to ℓ_n. It is recursively defined by the product of the probability of taking a transition from ℓ_i to ℓ_{i+1} within time interval I_i (cf. $(\star)$ and (1)) and the probability of the suffix cylinder set from ℓ_{i+1} and η_{i+1} on (cf. $(\star\star)$).

Schedulers. MTA incorporate nondeterministic decisions, which are resolved by schedulers (a.k.a, adversaries, policies, strategies, etc). For deciding which of the next action to take, generally a scheduler may "have access" to the current state only (memoryless/positional schedulers), or to the path from the initial to the current state (history dependent schedulers). In this paper, we are mainly interested in maximizing reachability probabilities. To this purpose, it is not difficult to see that, at each *state*, the scheduler can prescribe the decision solely depending on the current state instead of the history. Namely, only positional schedulers suffice. We stress that the decision is made at the level of *states* instead of *locations*, which implies that both the location and the clock valuation are relevant. Moreover, for time-bounded reachability probabilities, the global time affects the decision. (It can be considered as an extra clock which is never reset; cf. Section 5.) Hence, we consider a subclass of schedulers, namely, the *total time positional* (TTP) deterministic schedulers [NSK09], which guarantee the optimal solutions in the current setting (cf. [Ver85] for details).

Definition 6. *A* TTP scheduler *for an* MTA $\mathcal{M}$ *is a measurable function* $u{:}Loc{\times}\mathbb{R}_{\geq 0}^{|\mathcal{X}|}\times\mathbb{R}_{\geq 0}{\to}Act$ *such that for any location* ℓ*, clock valuation* η *and time* t*,* $u(\ell,\eta,t){\in}\mathcal{I}(\ell,\eta)$*.*

We denote $\mathcal{U}$ as the set of all TTP schedulers. Given $\mathcal{M}$ and a scheduler u, one can obtain a Markov process (with continuous state space) in the standard way.

Problem statement. In this paper, we are mainly interested in maximizing reachability probabilities in an MTA $\mathcal{M}$. There are two variants: *time-bounded reachability* and *unbounded reachability.* The former asks, given $\mathcal{M}$, a finite set of goal locations $Loc_F \subseteq Loc$ and a time bound $T \in \mathbb{R}_{\geq 0}$, what is the maximal probability to reach Loc_F, under any TTP scheduler $\mathcal{U}$ within T time units, i.e., $\sup_{u\in\mathcal{U}}\big\{Prob^u\big((\ell_0,0),\Diamond^{\leqslant T}Loc_F\big)\big\}$. The latter asks the same question except that there is no time constraint to reach Loc_F, i.e., $\sup_{u\in\mathcal{U}}\{Prob^u((\ell_0,0),\Diamond Loc_F)\}$. Here, the measure $Prob^u$ is the extension of $\Pr^{\mathcal{M}}$ which defines the probability of cylinder sets C under a given scheduler u.

Region construction for MTA*.* A main step in computing the maximum reachability probability is to transform the MTA into a region graph by means of the *region construction* [AD94], which allows us to obtain a PDDP from an MTA in a natural way.

As usual, a region is a constraint. For regions $\Theta,\Theta' \in \mathcal{B}(\mathcal{X})$, Θ' is the *successor region* of Θ if for all $\eta \models \Theta$ there exists $\delta \in \mathbb{R}_{>0}$ such that $\eta+\delta \models \Theta'$ and for all $\delta' < \delta$, $\eta+\delta' \models \Theta \vee \Theta'$. A region Θ *satisfies* a guard g (denoted $\Theta \models g$) iff $\forall \eta \models \Theta.\ \eta \models g$. A *reset operation* on region Θ is defined as $\Theta[X := 0] := \{\eta[X := 0] \mid \eta \models \Theta\}$. For any n-ary tuple J, let $J{\downharpoonright_i}$ denote the i-th entry in J, for $1 \leqslant i \leqslant n$.

Definition 7 (Region graph of MTA). *Given an* MTA $\mathcal{M} = (Act,\mathcal{X},Loc,\ell_0,E,\rightsquigarrow)$*, the region graph is* $\mathcal{G}(\mathcal{M}) = (Act,V,v_0,V_F,\Lambda,\hookrightarrow)$*, where* $V := Loc\times\mathcal{B}(\mathcal{X})$ *is a finite set of* vertices*, consisting of a location* ℓ *in* $\mathcal{M}$ *and a region* Θ*;* $v_0 \in V$ *is the* initial vertex *if* $(\ell_0,\vec{0}) \in v_0$*;* $\Lambda : V \to \mathbb{R}_{\geqslant 0}$ *is the* exit rate function *where* $\Lambda(v) := E(v{\downharpoonright_1})$*; and* $\hookrightarrow \subseteq V\times\big((Act{\times}[0,1]{\times}2^{\mathcal{X}}) \cup \{\delta\}\big)\times V$ *is the* transition (edge) relation*, such that:* - $v \overset{\delta}{\hookrightarrow} v'$ *is a delay transition if* $v{\downharpoonright_1}{=}v'{\downharpoonright_1}$ *and* $v'{\downharpoonright_2}$ *is a successor region of* $v{\downharpoonright_2}$*;*

- $v \overset{\alpha,p,X}{\hookrightarrow} v'$ *is a Markovian transition if there exists some transition* $v{\downharpoonright_1} \overset{\alpha,g}{\rightsquigarrow} \zeta$*,* $\zeta(X,v'{\downharpoonright_1}) = p$ *in* $\mathcal{M}$ *such that* $v{\downharpoonright_2} \models g$ *and* $v{\downharpoonright_2}[X := 0] \models v'{\downharpoonright_2}$*.*

The following result asserts that the region graph obtained from an MTA is in fact a PDDP. Moreover, any given MTA and its associated PDDP have the same reachability probabilities. Hence, the problem of computing reachability probabilities for MTA can be reduced to analyzing its corresponding PDDP.

Theorem 1. *Given any* MTA $\mathcal{M}$*, the region graph* $\mathcal{G}(\mathcal{M})$ *of* $\mathcal{M}$ *induces a* PDDP $\mathcal{Z}(\mathcal{M})$*. Moreover,* $\mathcal{M}$ *and* $\mathcal{Z}(\mathcal{M})$ *have the same maximal time-bounded/unbounded reachability probabilities.*

Proof. Let MTA $\mathcal{M} = (Act,Loc,\mathcal{X},\ell_0,E,\rightsquigarrow)$ with region graph $\mathcal{G}(\mathcal{M}) = (Act,V,v_0,\Lambda,\hookrightarrow)$. Define $\mathcal{Z}(\mathcal{M}) = (Act,V,\mathcal{X},Inv,\phi,\Lambda,\mu)$ where for any $v \in V$: $Inv(v) := v{\downharpoonright_2}$ and the state space $\mathbb{S} := \big\{(v,\eta) \mid v \in V, \eta \models Inv(v)\big\}$ is defined in the standard way; $\phi(v,\eta,t) := \eta+t$; $\Lambda(v,\eta) := \Lambda(v)$; for each delay transition $v \overset{\delta}{\hookrightarrow} v'$ in $\mathcal{G}(\mathcal{M})$ and any $\alpha \in Act$ we have $\mu(\xi,\alpha,\{\xi'\}) := 1$, where $\xi = (v,\eta)$, $\xi' = (v',\eta)$ and $\eta \models \partial Inv(v)$; for each Markovian transition $v \overset{\alpha,p,X}{\hookrightarrow} v'$ in $\mathcal{G}(\mathcal{M})$ we have $\mu(\xi,\alpha,\{\xi'\}) := p$, where

$\xi = (v, \eta)$, $\eta \models \mathit{Inv}(v)$ and $\xi' = (v', \eta[X := 0])$. It follows directly that $\mathcal{Z}(\mathcal{M})$ is a PDDP.

The time-unbounded reachability probability $\Pr(\ell, \eta)$ for an MTA $\mathcal{M}$ can be described by the following system of integral equations ($\ell \notin \mathit{Loc}_F$)

$$\Pr(\ell,\eta) = \max_{\alpha \in \mathcal{I}(\ell)} \left\{ \int_0^\infty \Lambda(\ell) e^{-\Lambda(\ell)\tau} \cdot \sum_{\ell \xmapsto[p,X]{\alpha,g} \ell'} \mathbf{1}_g(\eta+\tau) \cdot p \cdot \Pr(\ell', \eta')\, d\tau \right\},$$

where $\mathcal{I}(\ell)$ is the set of available actions in location ℓ, transition $\ell \xmapsto[p,X]{\alpha,g} \ell'$ is defined by the transition $\ell \overset{\alpha,g}{\rightsquigarrow} \zeta$, $\zeta(X, \ell') = p$, the characteristic function $\mathbf{1}_g(\eta+\tau)$ is as in Eq.(2) and $\eta' = (\eta + \tau)[X := 0]$. For $\ell \in \mathit{Loc}_F$ we define $\Pr(\ell, \eta) = 1$.

From here on we will consider that the clock constraints are of the form $x \unlhd c$, where $c \in \mathbb{N}_{\geqslant 0}$ and $\unlhd \in \{\leq, <, \geq, >\}$. For a transition $\ell \xmapsto[p,X]{\alpha,g} \ell'$ with guard g and clock valuation η we get that

$$\Pr(\ell,\eta) = \max_{\alpha \in \mathcal{I}(\ell)} \left\{ \int_{t_1}^{t_2} \Lambda(\ell) e^{-\Lambda(\ell)\tau} \cdot \sum_{\ell \xmapsto[p,X]{\alpha,g} \ell'} p \cdot \Pr(\ell', \eta')\, d\tau \right\},$$

where $\eta + \tau \models g$ and $\tau \in]t_1, t_2[$, $t_1, t_2 \in \mathbb{Q}_{\geqslant 0} \cup \{\infty\}$.

Given the region graph $\mathcal{Z}(\mathcal{M})$ of the MTA $\mathcal{M}$, let $\mathit{Prob}_v(\eta)$ be the maximum probability to reach the set of goal vertices V_F starting from vertex v and clock valuation η. $\mathit{Prob}_v(\eta)$ can be defined as:

$$\begin{aligned}
\mathit{Prob}_v(\eta) &= \begin{cases} \max_{\alpha \in \mathcal{I}(v\downharpoonright_1)} \{\mathit{Prob}_{v,\delta}(\eta) + \mathit{Prob}_{v,\alpha}(\eta)\}, & \text{if } v \neq V_F \\ 1, & \text{otherwise} \end{cases} \\
\mathit{Prob}_{v,\alpha}(\eta) &= \int_0^{\flat(v,\eta)} \Lambda(v) \cdot e^{-\Lambda(v)\tau} \cdot \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} p \cdot \mathit{Prob}_{v'}\big((\eta+\tau)[X:=0]\big)\, d\tau, \\
\mathit{Prob}_{v,\delta}(\eta) &= e^{-\Lambda(v)\flat(v,\eta)} \cdot \mathit{Prob}_{v'}\big(\eta + \flat(v,\eta)\big).
\end{aligned}$$

Here $\mathit{Prob}_{v,\alpha}(\eta)$ denotes the probability to reach V_F by taking a Markovian jump and $\mathit{Prob}_{v,\delta}(\eta)$ the probability to reach V_F through vertex v' by taking the boundary jump $v \overset{\delta}{\hookrightarrow} v'$.

For $\ell \notin \mathit{Loc}_F$ we have to prove that

$$\Pr(\ell, \eta) = \mathit{Prob}_{v_0}(\eta), \tag{3}$$

where v_0 is the initial vertex in the region graph $\hat{\mathcal{Z}}(\mathcal{M})$ induced by location ℓ such that $v_0\downharpoonright_1 = \ell$. In order to show the validity of Eq.(3) we define $\Pr^n(\ell, \eta)$ as the maximum time-unbounded reachability probability to reach the set of goal states Loc_F in n-MTA transition steps

$$\Pr^n(\ell,\eta) = \max_{\alpha \in \mathcal{I}(\ell)} \left\{ \int_{t_1}^{t_2} \Lambda(\ell) e^{-\Lambda(\ell)\tau} \cdot \sum_{\ell \xmapsto[p,X]{\alpha,g} \ell'} p \cdot \Pr^{n-1}(\ell', \eta')\, d\tau \right\}.$$

In the same way we define $Prob_v^n(\eta)$ as the maximum reachability probability to reach the set of goal states V_F in n-MTA transition steps

$$Prob_v^n(\eta) = \begin{cases} \max_{\alpha\in\mathcal{I}(v\downharpoonright_1)} \left\{Prob_{v,\delta}^n(\eta) + Prob_{v,\alpha}^n(\eta)\right\}, & \text{if } v \neq V_F \\ 1, & \text{otherwise} \end{cases}$$

$$Prob_{v,\alpha}^n(\eta) = \int_0^{\flat(v,\eta)} \Lambda(v)\cdot e^{-\Lambda(v)\tau}\cdot \sum_{v \xrightarrow{\alpha,p,X} v'} p\cdot Prob_{v'}^{n-1}\big((\eta+\tau)[X{:=}0]\big)\, d\tau,$$

$$Prob_{v,\delta}^n(\eta) = e^{-\Lambda(v)\flat(v,\eta)} \cdot Prob_{v'}^n\big(\eta + \flat(v,\eta)\big).$$

Now the task is to show that for all $n \in \mathbb{N}_{\geqslant 0}$,

$$\Pr{}^n(\ell,\eta) = Prob_{v_0}^n(\eta). \tag{4}$$

Notice that $\lim_{n\to\infty} \Pr^n(\ell,\eta) = \Pr(\ell,\eta)$ and $\lim_{n\to\infty} Prob_v^n(\eta) = Prob_v(\eta)$.

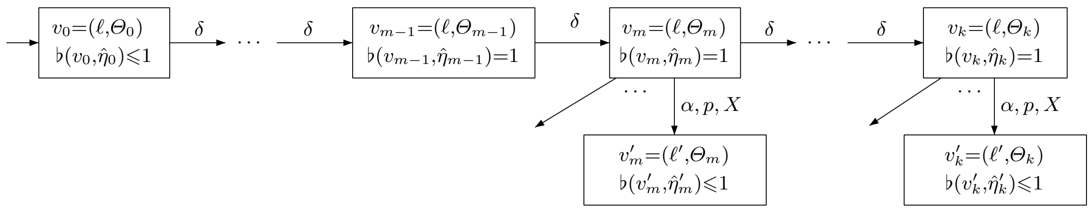

Fig. 2. The sub-region graph $\hat{\mathcal{Z}}(\mathcal{M})$ for the transition from ℓ to ℓ'

We will show the validity of Eq.(4) by induction on n.

- For the base case $n = 0$, we have that $\Pr^0(\ell,\eta) = 0$ and $Prob_{v_0}^0(\eta) = 0$ as $\ell \notin Loc_F$.
- For the inductive case we have to show the validity of Eq.(4) for $n{+}1$. We consider the MTA transition $\ell \overset{\alpha,g}{\rightsquigarrow} \zeta$ and its corresponding region graph $\hat{\mathcal{Z}}(\mathcal{M})$ shown in Fig. 2. For simplicity we consider that location ℓ induces the vertices $\{v_i = (\ell,\Theta_i) \mid 0 \leqslant i \leqslant k\}$. Note that for Markovian transitions, the regions stay the same. We denote $\hat{\eta}_i$ as the entering clock valuation in vertex v_i, for i the indices of the regions. Here $\hat{\eta}_0 = \eta$ and $\hat{\eta}_i = \hat{\eta}_{i-1} + \flat(v_{i-1},\hat{\eta}_{i-1})$ for $1 \le i \le k$. For any $\hat{\eta} \in \bigcup_{i=0}^{m-1} \Theta_i \cup \bigcup_{i>k} \Theta_i$, $\hat{\eta} \not\models g$; or more specifically,

  $$t_1 = \sum_{i=0}^{m-1} \flat(v_i,\hat{\eta}_i) \qquad \text{and} \qquad t_2 = \sum_{i=0}^{k} \flat(v_i,\hat{\eta}_i).$$

  For notation simplicity we define

  $$\Xi_v^n(\eta) = \max_{\alpha\in\mathcal{I}(v\downharpoonright_1)} \left\{Prob_{v,\delta}^n(\eta) + Prob_{v,\alpha}^n(\eta)\right\}.$$

  Given the fact that from v_0 the process can only execute a delay transition before time t_1, it holds that

  $$\Xi_{v_0}^{n+1}(\eta) = e^{-t_1\Lambda(v_0)} \cdot \Xi_{v_m}^{n+1}(\hat{\eta}_m),$$
  $$\Xi_{v_m}^{n+1}(\hat{\eta}_m) = \max_{\alpha\in\mathcal{I}(v_m\downharpoonright_1)} \left\{Prob_{v_m,\delta}^{n+1}(\hat{\eta}_m) + Prob_{v_m,\alpha}^{n+1}(\hat{\eta}_m)\right\}.$$

Notice that $\Lambda(v_0) = \Lambda(v_i)$ for all $i \leq k$. Therefore, by substitution we obtain:

$$\begin{aligned}
\Xi_{v_0}^{n+1}(\eta) &= e^{-t_1 \Lambda(v_0)} \cdot Prob_{v_m,\delta}^{n+1}(\hat{\eta}_m) + e^{-t_1 \Lambda(v_0)} \cdot \max_{\alpha \in \mathcal{I}(v_m \downharpoonleft_1)} \left\{ Prob_{v_m,\alpha}^{n+1}(\hat{\eta}_m) \right\} \\
&= e^{-t_1 \Lambda(v_0)} \cdot Prob_{v_m,\delta}^{n+1}(\hat{\eta}_m) \\
&+ e^{-t_1 \Lambda(v_0)} \cdot \int_0^{\flat(v_m,\hat{\eta}_m)} \Lambda(v_m) \cdot e^{-\Lambda(v_m)\tau} \cdot \max_{\alpha \in \mathcal{I}(v_m \downharpoonleft_1)} \left\{ \sum_{v_m \overset{\alpha,p,X}{\hookrightarrow} v_m'} p \cdot Prob_{v_m'}^{n} \big((\hat{\eta}_m + \tau)[X := 0] \big) \right\} d\tau \\
&= e^{-t_1 \Lambda(v_0)} \cdot Prob_{v_m,\delta}^{n+1}(\hat{\eta}_m) \\
&+ \int_{t_1}^{t_1 + \flat(v_m,\hat{\eta}_m)} \Lambda(v_m) \cdot e^{-\Lambda(v_m)\tau} \cdot \max_{\alpha \in \mathcal{I}(v_m \downharpoonleft_1)} \left\{ \sum_{v_m \overset{\alpha,p,X}{\hookrightarrow} v_m'} p \cdot Prob_{v_m'}^{n} \big((\hat{\eta}_m + \tau - t_1)[X := 0] \big) \right\} d\tau.
\end{aligned}$$

Evaluating each term $Prob_{v_m,\delta}^{n+1}(\hat{\eta}_m)$ we get the following sum of integrals:

$$\begin{aligned}
\Xi_{v_0}^{n+1}(\eta) = \sum_{i=0}^{k-m} \int_{t_1 + \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j})}^{t_1 + \sum_{j=0}^{i} \flat(v_{m+j},\hat{\eta}_{m+j})} & \Lambda(v_{m+i}) \cdot e^{-\Lambda(v_{m+i})\tau} \\
\cdot \max_{\alpha \in \mathcal{I}(v_m \downharpoonleft_1)} & \left\{ \sum_{v_{m+i} \overset{\alpha,p,X}{\hookrightarrow} v_{m+i}'} p \cdot Prob_{v_{m+i}'}^{n} \Big((\hat{\eta}_{m+i} + \tau - t_1 - \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}))[X := 0] \Big) \right\} d\tau.
\end{aligned}$$

Notice that $\mathcal{I}(v_m \downharpoonleft_1) = \mathcal{I}(v_{m+i} \downharpoonleft_1)$ for all $i \leq k$. Now we define the function $F_\alpha^n(t) : [t_1, t_2] \to [0,1]$, such that when $t \in [t_1 + \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}), t_1 + \sum_{j=0}^{i} \flat(v_{m+j},\hat{\eta}_{m+j})]$ for $i \leqslant k - m$ then

$$F_\alpha^n(t) = \sum_{v_{m+i} \overset{\alpha,p,X}{\hookrightarrow} v_{m+i}'} p \cdot Prob_{v_{m+i}'}^{n} \Big((\hat{\eta}_{m+i} + t - t_1 - \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}))[X := 0] \Big)$$

Using $F_\alpha^n(t)$ we can rewrite $\Xi_{v_0}^{n+1}(\eta)$ to an equivalent form as:

$$\Xi_{v_0}^{n+1}(\eta) = \int_{t_1}^{t_2} \Lambda(v_0) \cdot e^{-\Lambda(v_0)\tau} \cdot \max_{\alpha \in \mathcal{I}(v_m \downharpoonleft_1)} \{ F_\alpha^n(\tau) \} \, d\tau. \tag{5}$$

Here notice that

$$\hat{\eta}_{m+i} = \eta + \sum_{j=0}^{m-1} \flat(v_j, \hat{\eta}_j) + \sum_{j=0}^{i-1} \flat(v_{m+j}, \hat{\eta}_{m+j}).$$

Therefore, for any $t \in [t_1 + \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}), t_1 + \sum_{j=0}^{i} \flat(v_{m+j},\hat{\eta}_{m+j})]$, $i \leqslant k - m$ we obtain

$$\hat{\eta}_{m+i} + t - t_1 - \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}) = \eta + t.$$

From the I.H. we know that $\Pr^n(\ell, \eta) = Prob_{v_0}^{n}(\eta)$ such that $v_0 \downharpoonleft_1 = \ell$. Therefore, for any $t \in [t_1 + \sum_{j=0}^{i-1} \flat(v_{m+j},\hat{\eta}_{m+j}), t_1 + \sum_{j=0}^{i} \flat(v_{m+j},\hat{\eta}_{m+j})]$ and $v_{m+i}' \downharpoonleft_1 = \ell'$,

$i \leqslant k-m$, we get

$$\begin{aligned}
F^n_\alpha(t) &= \sum_{v_{m+i} \overset{\alpha,p,X}{\hookrightarrow} v'_{m+i}} p \cdot Prob^n_{v'_{m+i}}\big((\hat{\eta}_{m+i} + t - t_1 - \sum_{j=0}^{i-1} \flat(v_{m+j}, \hat{\eta}_{m+j}))[X := 0]\big) \\
&= \sum_{v_{m+i} \overset{\alpha,p,X}{\hookrightarrow} v'_{m+i}} p \cdot Prob^n_{v'_{m+i}}\big((\eta + t))[X := 0]\big) \\
&= \sum_{v_{m+i} \overset{\alpha,p,X}{\hookrightarrow} v'_{m+i}} p \cdot \Pr^n(\ell', (\eta + t))[X := 0]) \\
&= \sum_{\ell \xmapsto[p,X]{\alpha,g} \ell'} p \cdot \Pr^n(\ell', (\eta + t))[X := 0]).
\end{aligned}$$

Eq.(5) results in

$$\varXi^{n+1}_{v_0}(\eta) = \int_{t_1}^{t_2} \varLambda(\ell) \cdot e^{-\varLambda(\ell)\tau} \cdot \max_{\alpha \in \mathcal{I}(\ell)} \left\{ \sum_{\ell \xmapsto[p,X]{\alpha,g} \ell'} p \cdot \Pr^n(\ell', (\eta + \tau))[X := 0]) \right\} d\tau.$$

As $Prob^{n+1}_{v_0}(\eta) = \varXi^{n+1}_{v_0}(\eta)$ (for $v_0\lfloor_1 \notin Loc_F$) we get that $Prob^{n+1}_{v_0}(\eta) = \Pr^{n+1}(\ell, \eta)$. □

4 Time-bounded reachability

In this section, we concentrate on maximizing *time-bounded reachability probabilities* in a PDDP $\mathcal{Z}(\mathcal{M}) = (Act, V, \mathcal{X}, Inv, \phi, \varLambda, \mu)$, namely, given a set V_F of goal locations and time bound T, we are interested in maximizing the probability to reach V_F within T time units. To this end, we first transform the PDDP by adding $t \leqslant T$ to each of its invariants. Namely, the global time t is read as an extra clock which is initialized to zero in the beginning and never reset. Let $\flat(v, \eta, t)$ be the minimal time for state (v, η) to hit the boundary $\partial Inv(v)$ at time t.

The following Bellman (dynamic programming) equations [CD88] for continuous state spaces play an essential role in solving the time-bounded reachability problem. Let $P(v, \eta, t)$ be the maximal probability for state (v, η) to reach V_F within time bound T at time t. $P(v, \eta, t) = 1$ if $v \in V_F$ and $t \leqslant T$, 0 if $t > T$; and otherwise

$$\begin{aligned}
P(v, \eta, t) = \max_{\alpha \in \mathcal{I}(v)} \Bigg\{ & \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^{\flat(v,\eta,t)} \varLambda(v) e^{-\varLambda(v)\tau} P(v', \eta', t{+}\tau) \cdot p \; d\tau \Bigg\} \\
& + e^{-\varLambda(v)\flat(v,\eta,t)} P(v'', \eta + \flat(v, \eta, t), t + \flat(v, \eta, t))
\end{aligned} \tag{6}$$

where $\mathcal{I}(v)$ is the set of actions enabled in v, $\eta' = (\eta + \tau)[X := 0]$ and $v \overset{\delta}{\hookrightarrow} v''$, where v'' is the time successor of v. The first summand represents the maximum reachability probability (among all the enabled actions) by taking a Markovian jump $v \overset{\alpha,p,X}{\hookrightarrow} v'$ and the other summand represents the probability of taking the boundary jump $v \overset{\delta}{\hookrightarrow} v''$.

We will provide two ways to solve (6): one by the discretization of (6) and the other based on the Hamilton-Jacobi-Bellman equation which is a partial differential equation.

4.1 Discretization

Our first approach is to discretize the continuous variables in the Bellman equation. Using a discretization step $h = \frac{1}{N}$ ($N \in \mathbb{N}_{>0}$), the aim is to obtain a *finite state* MDP $\mathcal{D}(\mathcal{M})$ from the PDDP $\mathcal{Z}(\mathcal{M})$. For this MDP, a similar Bellman equation can be derived and solved efficiently e.g. by value iteration [Ber95]. Intuitively, h is the lengh of time in which a *single* Markovian jump takes place from a given location.

Lemma 1. *For any discretization step h, $P(v, \eta, t)$ can be characterized as follows:*

$$P(v,\eta,t) = \begin{cases} \max_{\alpha\in\mathcal{I}(v)} \left\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \int_0^h \Lambda(v) e^{-\Lambda(v)\tau} P(v',\eta',t+\tau)\cdot p \; d\tau \right\} \\ \qquad +e^{-\Lambda(v)h} P(v,\eta+h,t+h), \qquad \textit{if } h < \flat(v,\eta,t) \\ e^{-\Lambda(v)\flat(v,\eta,t)} P(v'',\eta{+}\flat(v,\eta,t),t{+}\flat(v,\eta,t)), \qquad \textit{o.w.} \end{cases} \tag{7}$$

Proof. We consider the following two cases:

Case $h \geqslant \flat(v,\eta,t)$. We get that $P(v,\eta,t)$ is determined by the probability to take the delay transition $v \xhookrightarrow{\delta} v''$. Notice that as soon as $h \geqslant \flat(v,\eta,t)$ holds the boundary $\partial Inv(v)$ is hit and therefore a delay transition is taken. We obtain that

$$P(v,\eta,t) = e^{-\Lambda(v)\flat(v,\eta,t)} P(v'',\eta{+}\flat(v,\eta,t),t{+}\flat(v,\eta,t));$$

Case $h < \flat(v,\eta,t)$. We get

$$\begin{aligned}
P(v,\eta,t) &= \max_{\alpha\in\mathcal{I}(v)} \Bigg\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \int_0^{\flat(v,\eta,t)} p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau + \\
&\quad e^{-\Lambda(v)\flat(v,\eta,t)} P(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)) \Bigg\} \\
&= \max_{\alpha\in\mathcal{I}(v)} \Bigg\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \left(\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau + \int_h^{\flat(v,\eta,t)} p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau \right) \\
&\quad + e^{-\Lambda(v)\flat(v,\eta,t)} P(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)) \Bigg\} \\
&= \max_{\alpha\in\mathcal{I}(v)} \Bigg\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \left(\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau + e^{-\Lambda(v)h}\int_0^{\flat(v,\eta,t)-h} p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta'+h,t{+}h{+}\tau)d\tau \right) \\
&\quad + e^{-\Lambda(v)\flat(v,\eta,t)} P(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)) \Bigg\} \\
&= \max_{\alpha\in\mathcal{I}(v)} \Bigg\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau + \\
&\quad e^{-\Lambda(v)h} \int_0^{\flat(v,\eta+h,t+h)} p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta'+h,t{+}h{+}\tau)d\tau + \\
&\quad e^{-\Lambda(v)(\flat(v,\eta,t)-h)} e^{-\Lambda(v)h} P(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)) \Bigg\} \\
&= \max_{\alpha\in\mathcal{I}(v)} \Bigg\{ \sum_{v \xhookrightarrow{\alpha,p,X} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau + \\
&\quad e^{-\Lambda(v)h} \Bigg(\int_0^{\flat(v,\eta+h,t+h)} p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta'+h,t{+}h{+}\tau)d\tau +
\end{aligned}$$

$$e^{-\Lambda(v)\flat(v,\eta+h,t+h)}P(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t))\Big)\Bigg\}$$

$$= \max_{\alpha\in\mathcal{I}(v)}\left\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}P(v',\eta',t{+}\tau)d\tau\right\}+e^{-\Lambda(v)h}P(v,\eta+h,t+h).$$

This lemma states that the two characterizations ((6) and (7)) of the Bellman equation coincide. Specifically, the case that a Markovian jump takes place from the interval $[h,\flat(v,\eta,t))$ in (6) is captured by the term $e^{-\Lambda(v)h}P(v,\eta+h,t+h)$ given that no transition is taken to leave v in $[0,h)$.

Using h, each Markovian jump in the PDDP can be approximated by a Markovian jump which only takes place at time point $\{0,1,\ldots,Nh\}$. This gives rise to an MDP:

Definition 8. *Given* PDDP $\mathcal{Z}(\mathcal{M}){=}(Act,V,\mathcal{X},Inv,\phi,\Lambda,\mu)$*, and a discretization step* $h{=}\frac{1}{N}$ *(*$N{\in}\mathbb{N}_{>1}$*), the resulting* MDP $\mathcal{D}_h(\mathcal{M}){=}(Act,S,s_0,\mathcal{P})$ *is as follows:* $S{=}\{(v,\eta,t)\mid v{\in}V\wedge\eta{\models}Inv(v)\wedge t{\leqslant}T\}$*;* $s_0=(v_0,\vec{0},0)$*; For each* (v,η,t) *we distinguish two cases: (i) If* $h{<}\flat(v,\eta,t)$ *and* $v\overset{\alpha,p,X}{\hookrightarrow}v'$ *then* $\mathcal{P}\big((v,\eta,t),\alpha,(v',(\eta{+}h)[X{:=}0]),t{+}h)\big){=}\ p{\cdot}(1{-}e^{-\Lambda(v)h})$*; (ii) If* $h{\geqslant}\flat(v,\eta,t)$ *and* $v\overset{\delta}{\hookrightarrow}v'$*, then* $\mathcal{P}\,((v,\eta,t),\alpha,(v',\eta{+}\flat(v,\eta,t),t{+}\flat(v,\eta,t)))={e^{-\Lambda(v)\flat(v,\eta,t)}}$.

For each state in $\mathcal{D}_h(\mathcal{M})$ there is an outgoing transition of type either (i) or (ii). Let $G(v,\eta,t)$ be the maximal reachability probability in the MDP $\mathcal{D}_h(\mathcal{M})$. $G(v,\eta,t)$ can be characterized as:

$$G(v,\eta,t){=}\begin{cases}\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}p(1-e^{-\Lambda(v)h})G(v',\hat{\eta}',t+h)\Big\}\\ \quad +e^{-\Lambda(v)h}G(v,\eta+h,t+h), \qquad \text{if } h<\flat(v,\eta,t)\\ e^{-\Lambda(v)\flat(v,\eta,t)}G(v'',\eta{+}\flat(v,\eta,t),t{+}\flat(v,\eta,t)), \qquad \textit{o.w.}\end{cases} \tag{8}$$

where $\hat{\eta}'=(\eta+h)[X:=0]$.

Lemma 2. *Given* PDDP $\mathcal{Z}(\mathcal{M})$ *and its approximated* MDP $\mathcal{D}_h(\mathcal{M})$ *with* $h=\frac{1}{N}$ *(*$N\in\mathbb{N}_{>0}$*), the state space of* $\mathcal{D}_h(\mathcal{M})$ *is finite and is of* $\mathcal{O}(|V|\cdot N^{(|\mathcal{X}|+1)})$.

Proof. On can easily see that in (8) the value of the global time t as well as of the clock valuation η is a multiple of $\frac{1}{N}$, i.e., $\frac{k}{N}$, $k\in\mathbb{N}$ and $k\le N$. Notice when t or η is $\frac{k}{N}$ then the time to hit the boundary $\flat(v,\eta,t)$ is also a multiple of $\frac{1}{N}$, i.e., $\frac{N-k}{N}$. Given a vertix $v\in V$ and its corresponding region $Inv(v)$ the number of discretization points induced by h is $\frac{N^{(|X|+1)}}{2}$, where $|X|+1$ represents the number of clocks $|\mathcal{X}|$ plus the global time and the denominator 2 is due to the fact that $Inv(v)$ represent a tetrahedron. □

Based on (7) we define two integral operators

$$\mathcal{F}:(V\times\mathcal{B}(\mathcal{X})\times\mathbb{R}_{\geqslant0}\to[0,1])\to(V\times\mathcal{B}(\mathcal{X})\times\mathbb{R}_{\geqslant0}\to[0,1]),$$
$$\tilde{\mathcal{F}}:(V\times\mathcal{B}(\mathcal{X})\times\mathbb{R}_{\geqslant0}\to[0,1])\to(V\times\mathcal{B}(\mathcal{X})\times\mathbb{R}_{\geqslant0}\to[0,1]),$$

where

$$(\mathcal{F}H)(v,\eta,t)=\begin{cases}\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}H(v',\eta',t+\tau)d\tau\Big\}+\\ \qquad e^{-\Lambda(v)h}H(v,\eta+h,t+h), \qquad \text{if } h<\flat(v,\eta,t)\\ e^{-\Lambda(v)\flat(v,\eta,t)}H(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)), \qquad \text{if } h\ge\flat(v,\eta,t)\end{cases}$$

and

$$(\tilde{\mathcal{F}}H)(v,\eta,t)=\begin{cases}\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}H(v',\tilde{\eta}',t)d\tau\Big\}+\\ \qquad e^{-\Lambda(v)h}H(v,\eta+h,t+h), \qquad \text{if } h<\flat(v,\eta,t)\\ e^{-\Lambda(v)\flat(v,\eta,t)}H(v'',\eta+\flat(v,\eta,t),t+\flat(v,\eta,t)), \qquad \text{if } h\ge\flat(v,\eta,t)\end{cases}$$

such that $\eta' = (\eta+\tau)[X := 0]$ and $\tilde{\eta}' = \eta[X := 0]$. The integral operators act on measurable functions $H : V \times \mathcal{B}(\mathcal{X}) \times \mathbb{R}_{\geqslant 0} \to [0,1]$ such that $(\mathcal{F}H)(v,\eta,t) = 1$ and $(\tilde{\mathcal{F}}H)(v,\eta,t) = 1$ if $v \in V_F$ and $t \leq T$.

Lemma 3. *For any measurable function $H : V \times \mathcal{B}(\mathcal{X}) \times \mathbb{R}_{\geqslant 0} \to [0,1]$ and the least fixpoint $G(v,\eta,t)$ for the equation $H(v,\eta,t) = (\tilde{\mathcal{F}}H)(v,\eta,t)$ we have $(\tilde{\mathcal{F}}H)(v,\eta,t) \geq G(v,\eta,t)$, where $v \in V$, $\eta \in v\lfloor_2$ and $t \leq T$.*

Proof. Let $H(v,\eta,t)$ be a fixpoint for the equation $H(v,\eta,t) = (\tilde{\mathcal{F}}H)(v,\eta,t)$ in order to show that $(\tilde{\mathcal{F}}H)(v,\eta,t) \geq G(v,\eta,t)$ we will show by induction on $n \in \mathbb{N}$ that $(\tilde{\mathcal{F}}H)(v,\eta,t) \geq G^n(v,\eta,t)$, where $\lim_{n\to\infty} G^n(v,\eta,t) = G(v,\eta,t)$.

- Base case: $G^0(v,\eta,t) = 1 = (\tilde{\mathcal{F}}H)(v,\eta,t)$ if $v \in V_F$ and $t \leq T$ and $G^0(v,\eta,t) = 0 \leq (\tilde{\mathcal{F}}H)(v,\eta,t)$, otherwise.
- Induction hypothesis: $G^n(v,\eta,t) \leq (\tilde{\mathcal{F}}H)(v,\eta,t)$.
- Induction step: for $h < \flat(v,\eta,t)$ we get

$$\begin{aligned}
G^{n+1}(v,\eta,t) &= (\tilde{\mathcal{F}}G^n)(v,\eta,t) \\
&= \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v) e^{-\Lambda(v)\tau} G^n(v',\tilde{\eta}',t) d\tau \Big\} + e^{-\Lambda(v)h} G^n(v,\eta+h,t+h) \\
\text{(I.H.)} &\leq \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v) e^{-\Lambda(v)\tau} H(v',\tilde{\eta}',t) d\tau \Big\} + e^{-\Lambda(v)h} H(v,\eta+h,t+h) \\
&= (\tilde{\mathcal{F}}H)(v,\eta,t).
\end{aligned}$$

We obtain that $(\tilde{\mathcal{F}}H)(v,\eta,t) \geq G^{n+1}(v,\eta,t)$ and $(\tilde{\mathcal{F}}H)(v,\eta,t) \geq \lim_{n\to\infty} G^n(v,\eta,t) = G(v,\eta,t)$. □

Lemma 4. *For the least fixpoints $P(v,\eta,t)$ and $G(v,\eta,t)$ given by the equations $P(v,\eta,t) = (\mathcal{F}P)(v,\eta,t)$ and $G(v,\eta,t) = (\tilde{\mathcal{F}}G)(v,\eta,t)$ we get that $G(v,\eta,t) \geq P(v,\eta,t)$, where $v \in V$, $\eta \in v\lfloor_2$ and $t \leq T$.*

Proof. We will prove that $P^n(v,\eta,t) - G^n(v,\eta,t) \leq 0$, by induction on n, where $\lim_{n\to\infty} P^n(v,\eta,t) = P(v,\eta,t)$ and $\lim_{n\to\infty} G^n(v,\eta,t) = G(v,\eta,t)$.

- Base case: $P^0(v,\eta,t) = 1 = G^0(v,\eta,t)$ if $v \in V_F$, and $t \leq T$ and $P^0(v,\eta,t) = 0 = G^0(v,\eta,t)$, otherwise.
- Induction hypothesis: $P^n(v,\eta,t) - G^n(v,\eta,t) \leq 0$.
- Induction step: for $h < \flat(v,\eta,t)$ we get

$$\begin{aligned}
&P^{n+1}(v,\eta,t) - G^{n+1}(v,\eta,t) = \\
&\max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v) e^{-\Lambda(v)\tau} P^n(v',\eta',t+\tau) d\tau \Big\} + e^{-\Lambda(v)h} P^n(v,\eta+h,t+h) - \\
&\max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v) e^{-\Lambda(v)\tau} G^n(v',\tilde{\eta}',t) d\tau \Big\} - e^{-\Lambda(v)h} G^n(v,\eta+h,t+h) \leq \\
&\max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v) e^{-\Lambda(v)\tau} (P^n(v',\eta',t+\tau) - G^n(v',\tilde{\eta}',t)) d\tau \Big\} + \\
&e^{-\Lambda(v)h} (P^n(v,\eta+h,t+h) - G^n(v,\eta+h,t+h)).
\end{aligned}$$

  From the definition of the maximum time-bounded reachability we know that $P(v,\eta,t)$ is the maximum probability to reach a set of goal states in $T-t$ units of time. Therefore, the function $P(v,\eta,t)$ is monotonously decreasing in its third argument, i.e., by increasing t the value of $P(v,\eta,t)$ decreases. As a result we get that $P(v',\tilde{\eta}',t) \geq P(v',\eta',t+\tau)$ for all $\tau \in [0,h]$. We obtain that $P^n(v',\tilde{\eta}',t) \geq P^n(v',\eta',t+\tau)$ for all $\tau \in [0,h]$ and:

$$
\begin{aligned}
&P^{n+1}(v,\eta,t) - G^{n+1}(v,\eta,t) \leq \\
&\max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}(P^n(v',\tilde{\eta}',t) - G^n(v',\tilde{\eta}',t))d\tau \Big\} + \\
&e^{-\Lambda(v)h}(P^n(v,\eta+h,t+h) - G^n(v,\eta+h,t+h)).
\end{aligned}
$$

From induction hypothesis we know that $P^n(v',\tilde{\eta}',t) - G^n(v',\tilde{\eta}',t) \leq 0$ and $P^n(v,\eta+h,t+h) - G^n(v,\eta+h,t+h) \leq 0$ as result we obtain $P^{n+1}(v,\eta,t) - G^{n+1}(v,\eta,t) \leq 0$, which proves the lemma.

□

Notice, by using the integral operator $\tilde{\mathcal{F}}$ on function $G(v,\eta,t)$ we obtain Eq. 8 for $h < \flat(v,\eta,t)$ as follows:

$$
\begin{aligned}
G(v,\eta,t) &= \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}G(v',\tilde{\eta}',t)d\tau \Big\} + e^{-\Lambda(v)h}G(v,\eta+h,t+h) \\
&= \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}d\tau \cdot G(v',\tilde{\eta}',t) \Big\} + e^{-\Lambda(v)h}G(v,\eta+h,t+h) \\
&= \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} p(1-e^{-\Lambda(v)h})G(v',\tilde{\eta}',t) \Big\} + e^{-\Lambda(v)h}G(v,\eta+h,t+h)
\end{aligned}
$$

Theorem 2 (Error bound). *For any state (v,η), time bound T, discretization step $h = \frac{1}{N}$ and $\lambda = \max_{v\in V}\{\Lambda(v)\}$: $\sup_{t\in[0,T]} |P(v,\eta,t) - G(v,\eta,t)| \leq (1-e^{-\lambda h})(1-e^{-\lambda T})$.*

Proof. By using the lemmas 3,4 we get that:

$$
\begin{aligned}
\sup_{t\in[0,T]} |P(v,\eta,t) - G(v,\eta,t)| &\leq \sup_{t\in[0,T]} |P(v,\eta,t) - (\tilde{\mathcal{F}}P)(v,\eta,t)| \\
&\leq \sup_{t\in[0,T]} |(\mathcal{F}P)(v,\eta,t) - (\tilde{\mathcal{F}}P)(v,\eta,t)|,
\end{aligned}
$$

where $P(v,\eta,t)$ is the least fixpoint of the equation $P(v,\eta,t) = (\mathcal{F}P)(v,\eta,t)$. Therefore, we have to show that

$$
\sup_{t\in[0,T]} \Big|(\mathcal{F}P)(v,\eta,t) - (\tilde{\mathcal{F}}P)(v,\eta,t)\Big| \leq (1-e^{-\lambda h})(1-e^{-\lambda T})
$$

for every vertex v and clock valuation η.

For the case $h \geq \flat(v,\eta,t)$ the theorem trivially holds as $(\mathcal{F}P)(v,\eta,t) - (\tilde{\mathcal{F}}P)(v,\eta,t) = 0$. On the other hand for $h < \flat(v,\eta,t)$ we have the following

$$
\begin{aligned}
&\sup_{t\in[0,T]} \Big|(\mathcal{F}P)(v,\eta,t) - (\tilde{\mathcal{F}}P)(v,\eta,t)\Big| \leq \\
&\sup_{t\in[0,T]} \left| \max_{\alpha\in\mathcal{I}(v)} \Big\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}(P(v',\eta',t+\tau) - P(v',\tilde{\eta}',t))d\tau \Big\} \right| \leq
\end{aligned}
$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}\sup_{t\in[0,T]}|P(v',\eta',t+\tau)-P(v',\tilde{\eta}',t)|\,d\tau\Big\}\leq$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}\sup_{t\in[0,T]}|P(v',\tilde{\eta}',t)|\,d\tau\Big\}\leq$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}\int_0^h p\Lambda(v)e^{-\Lambda(v)\tau}d\tau\sup_{t\in[0,T]}|P(v',\tilde{\eta}',t)|\Big\}\leq$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}p(1-e^{-\Lambda(v)h})\sup_{t\in[0,T]}|P(v',\tilde{\eta}',t)|\Big\}\leq$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}p(1-e^{-\Lambda(v)h})P(v',\tilde{\eta}',0)\Big\}\leq$$

$$\max_{\alpha\in\mathcal{I}(v)}\Big\{\sum_{v\overset{\alpha,p,X}{\hookrightarrow}v'}p(1-e^{-\Lambda(v)h})(1-e^{-\Lambda(v')T})\Big\}\leq(1-e^{-\lambda h})(1-e^{-\lambda T}),$$

where $\lambda=\max_{v\in V}\{\Lambda(v)\}$. Notice that in the above derivations we have used an upper bound for the maximum time-bounded rechability, i.e., $P(v',\tilde{\eta}',0)\leq(1-e^{-\Lambda(v')T})$. The upper bound represents the probability to reach a set of goal states in T units of time that are also all succesor states of v'. □

4.2 Hamilton-Jacobi-Bellman equations

As in traditional control theory [Ber95], the dynamic programming principles lead to a first-order integro-differential equation, which is the *Hamilton-Jacobi-Bellman* (HJB) *partial differential equation* (PDE).

For a PDDP $\mathcal{Z}(\mathcal{M})$ with the state space $\mathbb{S}$, we denote $P^u(v,\eta,t)$ as the maximal reachability probability from (v,η) and time t under scheduler u. Actually, $P^u(v,\eta,t)$ can be characterized by the expectation: $P^u(v,\eta,t)=\mathbb{E}\left[\mathbf{1}_{V_F}(Y^u(T))\mid Y^u(t)=(v,\eta)\right]$, where $Y^u(t)\in\mathbb{S}$ is the underlying stochastic process of $\mathcal{Z}(\mathcal{M})$ under scheduler u at time t and $\mathbf{1}_{V_F}(v,\eta)=1$ if $v\in V_F$ and 0 otherwise. To obtain the maximum time-bounded reachability probability, i.e., $\max_{u\in\mathcal{U}}\{P^u(v,\eta,t)\}$, we consider the following HJB equation with $f(v,\eta,t):=\max_{u\in\mathcal{U}}\{P^u(v,\eta,t)\}$, for every $(v,\eta)\in\mathbb{S}$ and $t\leqslant T$:

$$\frac{\partial f(v,\eta,t)}{\partial t}+\sum_{i=1}^{|\mathcal{X}|}\frac{\partial f(v,\eta,t)}{\partial\eta^{(i)}}=\max_{\alpha\in\mathcal{I}(v)}\left\{\Lambda(v)\sum_{v\overset{\alpha,X,p}{\hookrightarrow}v'}p\left(f(v,\eta,t)-f\left(v',\eta[X:=0],t\right)\right)\right\}\tag{9}$$

where $\eta^{(i)}$ is the i'th clock variable. The initial conditions of the above PDE are $f(v,\eta,t)=\mathbf{1}_{V_F}(v,\eta)$ for any $v\in V$ and $\eta\in Inv(v)$. Moreover, for every $\eta\in\partial Inv(v)$ and transition $v\overset{\delta}{\hookrightarrow}v'$, the boundary conditions take the form $f(v,\eta,t)=f(v',\eta,t)$.

Several methods can be used to solve the above HJB equation, e.g., the finite volume method [WJT03] or the time and state space discretization technique [Cam97].

Zero-clock. Following the same reasoning, for the CTMDPs, i.e., zero-clock MTAs, we can obtain similar results, except that instead of a system of PDEs we get a system of ODEs. As CTMDPs have zero clocks the resulting state space is finite. In (9) we have defined $f(v,\eta,t)$ as the maximum reachability probability. Given a *finite* state space, $f(v,\eta,t)$ can be simplified to $P_{i,j}(t)$, which is the probability to reach state ℓ_j at time T starting from state ℓ_i at time t. For any two states ℓ_i and ℓ_j we obtain the ODE: $\dfrac{dP_{i,j}(t)}{dt}=\max_{\alpha\in\mathcal{I}(\ell_i)}\left\{E_i\sum_k p_{i,k}(\alpha)\left(P_{i,j}(t)-P_{k,j}(t)\right)\right\}$, where $E_i:=E(\ell_i)$ and

$p_{i,k}(\alpha) := p$ such that $\ell_i \overset{\alpha,p}{\rightsquigarrow} \zeta_i$ and $\zeta_i(\varnothing, \ell_i) = \ell_k$. The system of ODEs can be also rewritten in the following matrix form:

$$\frac{d\hat{\mathbf{\Pi}}(t)}{dt} = -\max_{\alpha \in Act} \left\{ \mathbf{Q}(\alpha)\hat{\mathbf{\Pi}}(t) \right\}, \quad t \leqslant T, \tag{10}$$

where $\hat{\mathbf{\Pi}}(t)$ is the transition probability matrix at time t (the element (i,j) of $\hat{\mathbf{\Pi}}(t)$ is $P_{i,j}(t)$), $\hat{\mathbf{\Pi}}(T) = \mathbf{I}$, $\mathbf{Q}(\alpha) = \mathbf{R}(\alpha) - \mathbf{E}$ is the infinitesimal generator where $\mathbf{R}(\alpha)$ is the rate matrix (its element (i,j) is $E_i p_{i,j}(\alpha)$) and $\mathbf{E}$ is the exit rate matrix (all diagonal elements are the exit rates whereas the off-diagonal elements are zero).

A recent work [BS10] reveals that the above system of ODEs can be solved more efficiently than the general system of PDEs by adopting the adaptive uniformization technique.

5 Time-unbounded rechability

In this section, we focus on maximizing *unbounded reachability probabilities*. Namely, we are interested in maximizing the probability to eventually reach a given set of goal states. In contrast to the *time-bounded* case, there is no constraint on the time to reach the goal states. We also fix a PDDP $\mathcal{Z}(\mathcal{M}) = (Act, V, \mathcal{X}, Inv, \phi, \Lambda, \mu)$ first, and provide a Bellman (dynamic programming) equations, as follows. Let $P(v, \eta)$ be the maximal probability to reach V_F starting from vertex v and clock valuation η.

$$P(v,\eta) = \max_{\alpha \in \mathcal{I}(v)} \left\{ \sum_{v \overset{\alpha,p,X}{\hookrightarrow} v'} \int_0^{\flat(v,\eta)} p\Lambda(v) e^{-\Lambda(v)\tau} P(v', \eta') d\tau \right\} + e^{-\Lambda(v)\flat(v,\eta)} P(v'', \eta + \flat(v,\eta)) \tag{11}$$

where $\eta' = (\eta + \tau)[X := 0]$, $v \overset{\delta}{\hookrightarrow} v''$ and $P(v,\eta) = 1$ for $v \in V_F$.

Note that here, compared to (6), time does *not* play a role, i.e., the decision completely depends on the (continuous) state space. It might be difficult to solve (11), since the domain of the integration might be infinite. We proceed by considering some special cases, depending on the number of clocks in the model. The simplest case is CTMDP namely, the zero-clock MTA. This case is trivial since one can use the *embedded* MDP of the CTMDP and solve the reachability probability optimization problem. (Note that this is in contrast with the time-bounded case where the *embedded* MDP does not suffice.) We now move to the single-clock case.

For one-clock MTA, we can simplify the (general) Bellman equation (11) obtained before to a system of *linear* equations where the coefficients are either maximal time-bounded reachability probabilities for CTMDPs, which serve as a special case and have been solved in Section 4; or maximal time-unbounded reachability probabilities of CTMDPs, which by using the embedded MDP can be calculated quite efficiently.

Given an MTA $\mathcal{M}$, we denote the set of constants appearing in the clock constraints of $\mathcal{M}$ as $\{c_0, \ldots, c_m\}$ with $c_0 = 0$. We assume the following order: $0 = c_0 < c_1 < \cdots < c_m$. Let $\Delta c_i = c_{i+1} - c_i$ for $0 \leqslant i < m$. Note that for one clock MTA, regions in the region graph $\mathcal{G}(\mathcal{M})$ (cf. Section 3) can be represented by the following intervals: $[c_0, c_1), \ldots, [c_m, \infty)$. We partition the region graph $\mathcal{G}(\mathcal{M}) = (V, v_0, V_F, \Lambda, \hookrightarrow)$, or $\mathcal{G}$ for short, into a set of subgraphs

$$\mathcal{G}_i = (V_i, V_{Fi}, \Lambda_i, \{M_i^\alpha, B_i^\alpha, F_i\}_{\alpha \in Act}) \ ,$$

where $0 \leqslant i \leqslant m$ and $\Lambda_i(v) = \Lambda(v)$, if $v \in V_i$, 0 otherwise. These subgraphs are obtained by partitioning V, V_F and $\hookrightarrow$ as follows:

- $V = \bigcup_{0 \leqslant i \leqslant m} \{V_i\}$, where $V_i = \{(\ell, \Theta) \in V \mid \Theta \subseteq [c_i, c_{i+1})\}$;
- $V_F = \bigcup_{0 \leqslant i \leqslant m} \{V_{Fi}\}$, where $v \in V_{Fi}$ iff $v \in V_i \cap V_F$;

– $\hookrightarrow = \bigcup_{0 \leqslant i \leqslant m} \left(\bigcup_{\alpha \in Act} \{M_i^\alpha \cup B_i^\alpha\} \right) \cup F_i$, where for each $\alpha \in Act$, M_i^α is the set of *Markovian transitions (without reset)* between vertices inside $\mathcal{G}_i$ labeled by α; B_i^α is the set of *Markovian transitions (with reset)* from $\mathcal{G}_i$ to $\mathcal{G}_0$ (Backward) labeled by α; and F_i is the set of *delay transitions* from the vertices in $\mathcal{G}_i$ to that in $\mathcal{G}_{i+1}$ (Forward). It is easy to see that M_i^α, F_i, and B_i^α are pairwise disjoint.

Given a subgraph $\mathcal{G}_i$ ($0 \leqslant i \leqslant m$) with k_i states, define the probability vector $\vec{U}_i(x) = [u_i^1(x), \cdots, u_i^{k_i}(x)]^\top \in \mathbb{R}(x)^{k_i \times 1}$, where $u_i^j(x)$ is the maximal probability to go from vertex $v_i^j \in V_i$ to some vertex in V_F (in $\mathcal{G}$) at time point x. We now distinguish two cases:

Case $0 \leqslant i < m$. We first introduce some definitions as follows:

– $\mathbf{P}_i^{\alpha,\mathsf{M}} \in [0,1]^{k_i \times k_i}$ and $\mathbf{P}_i^{\alpha,\mathsf{B}} \in [0,1]^{k_i \times k_0}$ are probability transition matrices for Markovian and backward transitions respectively, parameterized by action α. Namely, for each vertex v and action $\alpha \in \mathcal{I}(v)$, $\mathbf{P}_i^{\alpha,\mathsf{M}}[v, v'] = p$, if $v \overset{\alpha,p,\varnothing}{\hookrightarrow} v'$; 0 otherwise. Similarly $\mathbf{P}_i^{\alpha,\mathsf{B}}[v, v'] = p$ if $v \overset{\alpha,p,\{x\}}{\hookrightarrow} v'$; 0 otherwise. Note that clearly we have that

$$\sum_{v'} \mathbf{P}_i^{\alpha,\mathsf{M}}(v, v') + \sum_{v''} \mathbf{P}_i^{\alpha,\mathsf{B}}(v, v'') = 1.$$

Moreover, we write

$$\mathbf{P}_i^\alpha = \left(\mathbf{P}_i^{\alpha,\mathsf{M}} \middle| \mathbf{P}_i^{\alpha,\mathsf{B}} \right) \ ,$$

and note $\mathbf{P}_i^\alpha \in [0,1]^{k_i \times (k_i + k_0)}$; and each row of $\mathbf{P}_i^\alpha$ sums up to 1.
– $\mathbf{D}_i(x) \in \mathbb{R}^{k_i \times k_i}$ is the delay probability matrix, i.e. for any $1 \leqslant j \leqslant k_i$, $\mathbf{D}_i(x)[j,j] = e^{-E(v_i^j)x}$. (The off diagonal elements are zero);
– $\mathbf{E}_i \in \mathbb{R}^{k_i \times k_i}$ is the exit rate matrix, i.e. for any $1 \leqslant j \leqslant k_i$, $\mathbf{E}_i[j,j] = E(v_i^j)$. (The off diagonal elements are zero);
– $\mathbf{M}_i^\alpha(x) = \mathbf{E}_i \cdot \mathbf{D}_i(x) \cdot \mathbf{P}_i^{\alpha,\mathsf{M}} \in \mathbb{R}^{k_i \times k_i}$ is the probability density matrix for the Markovian transitions inside $\mathcal{G}_i$ (i.e. for Markovian edges M_i^α); Namely, $\mathbf{M}_i^\alpha(x)[j,j']$ indicates the probability density function to take the Markovian jump *without* reset from the j-th vertex to the j'-th vertex in $\mathcal{G}_i$;
– $\mathbf{B}_i^\alpha(x) = \mathbf{E}_i \cdot \mathbf{D}_i(x) \cdot \mathbf{P}_i^{\alpha,\mathsf{B}} \in \mathbb{R}^{k_i \times k_0}$ is the probability density matrix for the reset edges B_i^a. Namely, $\mathbf{B}_i^\alpha(x)[j,j']$ indicates the probability density function to take the Markovian jump *with* reset from the j-th vertex in $\mathcal{G}_i$ to the j'-th vertex in $\mathcal{G}_0$; and
– $\mathbf{F}_i \in \mathbb{R}^{k_i \times k_{i+1}}$ is the incidence matrix for delay edges F_i. More specifically, $\mathbf{F}_i[j,j'] = 1$ indicates that there is a delay transition from the j-th vertex in $\mathcal{G}_i$ to the j'-th vertex in $\mathcal{G}_{i+1}$; 0 otherwise. Recall that for each action the delay edge is the same.

By instantiating the general Bellman equations, we obtain the following vector form

$$\begin{aligned} \vec{U}_i(x) = \ &\max_{\alpha \in Act} \Big\{ \underbrace{\int_0^{\Delta c_i - x} \mathbf{M}_i^\alpha(\tau) \vec{U}_i(x+\tau) d\tau}_{(*)} + \underbrace{\int_0^{\Delta c_i - x} \mathbf{B}_i^\alpha(\tau) d\tau \cdot \vec{U}_0(0)}_{(**)} + \\ &\mathbf{D}_i(\Delta c_i - x) \cdot \mathbf{F}_i \vec{U}_{i+1}(0) \Big\}, \quad x \in [0, \Delta c_i]. \end{aligned} \tag{12}$$

Let us explain the above equation. First of all, $\flat(v, x) = \Delta c_i - x$ for each state $1 \leqslant v \leqslant k_i$ in $\mathcal{G}_i$. The matrix $\mathbf{D}_i(\Delta c_i - x)$ indicates the probability to delay until the "end" of region i, and $\mathbf{F}_i \vec{U}_{i+1}(0)$ denotes the probability to continue in $\mathcal{G}_{i+1}$ (at relative time 0). In a similar way, the term $(*)$ reflects the case where clock x is not reset and the term $(**)$ considers the reset of x (and returning to $\mathcal{G}_0$).

Case $i = m$. $\vec{U}_m(x)$ is simplified as follows:

$$\vec{U}_m(x) = \max_{\alpha \in Act} \Big\{ \int_0^\infty \hat{\mathbf{M}}_m^\alpha(\tau)\vec{U}_m(x+\tau)d\tau + \tilde{\mathbf{1}}_F + \int_0^\infty \mathbf{B}_m^\alpha(\tau)d\tau \cdot \vec{U}_0(0) \Big\} \tag{13}$$

where $\hat{\mathbf{M}}_m^\alpha(\tau)[v,\cdot] = \mathbf{M}_m^\alpha(\tau)[v,\cdot]$ for $v \notin V_F$, 0 otherwise. $\tilde{\mathbf{1}}_F$ is a vector such that $\tilde{\mathbf{1}}_F[v] = 1$ if $v \in V_F$, 0 otherwise. Note that as the last subgraph $\mathcal{G}_m$ involves infinite regions, it has no delay transitions.

Before solving the system of integral equations (12)-(13), we first make the following observations:

(i) Due to the fact that inside $\mathcal{G}_i$ there are only Markovian jumps with neither resets nor delay transitions, $\mathcal{G}_i$ with (V_i, Λ_i, M_i) forms a CTMDP $\mathcal{C}_i$, say. For each $\mathcal{G}_i$ we define an *augmented* CTMDP $\mathcal{C}_i^\star$ with state space $V_i \cup V_0$, such that all V_0-vertices are made absorbing in $\mathcal{C}_i^\star$. The edges connecting V_i to V_0 are kept and all the edges inside $\mathcal{C}_0$ are removed. The augmented CTMDP is used to calculate the probability to start from a vertex in $\mathcal{G}_i$ and take a reset edge in a certain time.

(ii) Given any (finite state) CTMDP, for *time-bounded* reachability, we have the following equation (in matrix form).

$$\mathbf{\Pi}(x) = \max_{\alpha \in Act} \left\{ \int_0^x \widetilde{\mathbf{M}}^\alpha(\tau)\mathbf{\Pi}(x-\tau)d\tau + \mathbf{D}(x) \right\}. \tag{14}$$

where $\widetilde{\mathbf{M}}^\alpha(\tau)[v,v'] = e^{-E(v)\tau} \cdot p$ if there is a transition $v \overset{\alpha,\varnothing}{\rightsquigarrow} \zeta$ and $p = \zeta(\varnothing, v')$; 0 otherwise. Note that (14) is an instantiation of (6) and $\mathbf{\Pi}(T) = \hat{\mathbf{\Pi}}(0)$ (from Eq. (10)). Moreover, for augmented CTMDP $\mathcal{C}_i^\star$,

$$\widetilde{\mathbf{M}}^\alpha(\tau) = \left(\begin{array}{c|c} \mathbf{M}_i^\alpha(\tau) & \mathbf{B}_i^\alpha(\tau) \\ \mathbf{0} & \mathbf{I} \end{array}\right) .$$

Prior to exposing how to solve the system of integral equations by solving a system of *linear* equations by the next theorem, we define $\bar{\mathbf{\Pi}}_i^\star \in \mathbb{R}^{k_i \times k_0}$ for an augmented CTMDP $\mathcal{C}_i^\star$ to be part of $\mathbf{\Pi}_i^\star$, where $\bar{\mathbf{\Pi}}_i^\star$ only keeps the probabilities starting from V_i and ending in V_0. As a matter of fact,

$$\mathbf{\Pi}_i^\star(x) = \left(\begin{array}{c|c} \mathbf{\Pi}_i(x) & \bar{\mathbf{\Pi}}_i^\star(x) \\ \mathbf{0} & \mathbf{I} \end{array}\right),$$

where $\mathbf{0} \in \mathbb{R}^{k_0 \times k_i}$ is the matrix with all elements zero and $\mathbf{I} \in \mathbb{R}^{k_0 \times k_0}$ is the identity matrix.

Theorem 3. *For subgraph $\mathcal{G}_i$ of $\mathcal{G}$ with k_i states, it holds that:*

- *For $0 \leqslant i < m$, $\vec{U}_i(0) = \mathbf{\Pi}_i(\Delta c_i) \cdot \mathbf{F}_i \vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}_i^\star(\Delta c_i) \cdot \vec{U}_0(0)$, where $\mathbf{\Pi}_i(\Delta c_i)$ and $\bar{\mathbf{\Pi}}_i^\star(\Delta c_i)$ are for CTMDP $\mathcal{C}_i$ and the augmented CTMDP $\mathcal{C}_i^\star$, respectively.*
- *For $i = m$, $\vec{U}_m(0) = \max_{\alpha \in Act} \Big\{ \hat{\mathbf{P}}_m^\alpha \cdot \vec{U}_m(0) + \tilde{\mathbf{1}}_F + \hat{\mathbf{B}}_m^\alpha \cdot \vec{U}_0(0) \Big\}$, where $\hat{\mathbf{P}}_m^\alpha(v,v') = \mathbf{P}_m^\alpha(v,v')$ if $v \notin V_F$; 0 otherwise and $\hat{\mathbf{B}}_m^\alpha = \int_0^\infty \mathbf{B}_m^\alpha(\tau)d\tau$.*

Proof. We first deal with the case $i < m$. If in $\mathcal{G}_i$, for some action α there exists some backward edge, namely, for some j, j', $\mathbf{B}_i^\alpha(x)[j,j'] \neq 0$, then we shall consider the *augmented* CTMDP $\mathcal{C}_i^\star$ with $k_i^\star = k_i + k_0$ states. In view of this, the augmented integral equation $\vec{V}_i(x)$ is defined as:

$$\vec{V}_i^\star(x) = \max_{\alpha \in Act} \left\{ \int_0^{\Delta c_i - x} \mathbf{M}_i^{\alpha,\star}(\tau)\vec{V}_i^\star(x+\tau)d\tau + \mathbf{D}_i^\star(\Delta c_i - x) \cdot \mathbf{F}_i^\star \cdot \vec{\tilde{V}}_i(0) \right\},$$

where

– $\vec{V}_i^\star(x) = \left(\frac{\vec{V}_i(x)}{\vec{V}_i'(x)}\right) \in \mathbb{R}^{k_i^\star \times 1}$, where $\vec{V}_i'(x) \in \mathbb{R}^{k_0 \times 1}$ is the vector representing reachability probability for the augmented states in $\mathcal{G}_i$;
– $\mathbf{M}_i^{\alpha,\star}(\tau) = \left(\begin{array}{c|c}\mathbf{M}_i^\alpha(\tau) & \mathbf{B}_i^\alpha(\tau) \\ \hline \mathbf{0} & \mathbf{0}\end{array}\right) \in \mathbb{R}^{k_i^\star \times k_i^\star}$. For the augmented states, we assume that their exit rates are 0.
– $\mathbf{D}_i^\star(\tau) = \left(\begin{array}{c|c}\mathbf{D}_i(\tau) & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}\end{array}\right) \in \mathbb{R}^{k_i^\star \times k_i^\star}$.
– $\mathbf{F}_i^\star = \left(\mathbf{F}_i' \middle| \mathbf{B}_i'\right) \in \mathbb{R}^{k_i^\star \times (k_{i+1}+k_0)}$ such that $\mathbf{F}_i' = \left(\frac{\mathbf{F}_i}{\mathbf{0}}\right) \in \mathbb{R}^{k_i^\star \times k_{i+1}}$ is the incidence matrix for delay edges and $\mathbf{B}_i^{'} = \left(\frac{\mathbf{0}}{\mathbf{I}}\right) \in \mathbb{R}^{k_i^\star \times k_0}$, $\hat{\vec{V}}_i(0) = \left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \in \mathbb{R}^{(k_{i+1}+k_0)\times 1}$.

In the sequel, we shall prove two claims:

Claim 1. For each $0 \leqslant j \leqslant k_i$,

$$\vec{U}_i[j] = \vec{V}_i^\star[j] \ .$$

Proof of Claim 1. According to the definition, we have that

$$\vec{V}_i^\star(x) = \max_{\alpha \in Act} \left\{ \int_0^{\Delta c_i - x} \left(\begin{array}{c|c}\mathbf{M}_i^\alpha(\tau) & \mathbf{B}_i^\alpha(\tau) \\ \hline \mathbf{0} & \mathbf{0}\end{array}\right) \cdot \vec{V}_i^\star(x+\tau) d\tau + \left(\begin{array}{c|c}\mathbf{D}_i(\Delta c_i - x) & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}\end{array}\right) \cdot \left(\begin{array}{c|c}\mathbf{F}_i & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}\end{array}\right) \cdot \left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \right\}$$

It follows immediately that $\vec{V}_i'(x) = \vec{U}_0(0)$. For $\vec{V}_i(x)$, we have that

$$\begin{aligned}
&\vec{V}_i(x) \\
&= \max_{\alpha \in Act} \left\{ \int_0^{\Delta c_i - x} \mathbf{M}_i^\alpha(\tau) \vec{V}_i(x+\tau) d\tau + \int_0^{\Delta c_i - x} \mathbf{B}_i^\alpha(\tau) \vec{V}_i'(x+\tau) d\tau \right. \\
&\qquad \left. + \mathbf{D}_i(\Delta c_i - x) \cdot \mathbf{F}_i \cdot \vec{U}_{i+1}(0) \right\} \\
&= \max_{\alpha \in Act} \left\{ \int_0^{\Delta c_i - x} \mathbf{M}_i^\alpha(\tau) \vec{V}_i(x+\tau) d\tau + \int_0^{\Delta c_i - x} \mathbf{B}_i^\alpha(\tau) d\tau \cdot \vec{U}_0(0) \right. \\
&\qquad \left. + \mathbf{D}_i(\Delta c_i - x) \cdot \mathbf{F}_i \cdot \vec{U}_{i+1}(0) \right\} \\
&= \vec{U}_i(x)
\end{aligned}$$

Claim 2.

$$\vec{V}_i^\star(x) = \mathbf{\Pi}_i^\star(\Delta c_i - x) \cdot \mathbf{F}_i^\star \hat{\vec{V}}_i(0) \ ,$$

where

$$\mathbf{\Pi}_i^\star(x) = \max_{\alpha \in Act} \left\{ \int_0^x \mathbf{M}_i^{\alpha,\star}(\tau) \mathbf{\Pi}_i^\star(x-\tau) d\tau + \mathbf{D}_i^\star(x) \right\} \ .$$

Standard arguments yield that the optimal probability corresponds to the least fixpoint of a functional and can be computed iteratively from set $c_{i,x} = \Delta c_i - x$.

$$\begin{aligned}
\vec{V}_i^{\star,(0)}(x) &= \vec{0} \\
\vec{V}_i^{\star,(j+1)}(x) &= \max_{\alpha \in Act} \left\{ \int_0^{c_{i,x}} \mathbf{M}_i^\alpha(\tau) \vec{V}_i^{\star,(j)}(x+\tau) d\tau + \mathbf{D}_i^\star(c_{i,x}) \cdot \mathbf{F}_i^\star \hat{\vec{V}}_i(0) \right\}.
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{\Pi}_i^{\star,(0)}(c_{i,x}) &= \mathbf{0} \\
\mathbf{\Pi}_i^{\star,(j+1)}(c_{i,x}) &= \max_{\alpha\in Act}\left\{\int_0^{c_{i,x}} \mathbf{M}_i^{\star}(\tau)\mathbf{\Pi}_i^{\star,(j)}(c_{i,x}-\tau)d\tau + \mathbf{D}_i^{\star}(c_{i,x})\right\}.
\end{aligned}$$

By induction on j, we prove the following relation:

$$\vec{V}_i^{\star,(j)}(x) = \mathbf{\Pi}_i^{\star,(j)}(c_{i,x}) \cdot \mathbf{F}_i\hat{\vec{V}}_i(0) \ .$$

- Base case: $\vec{V}_i^{\star,(0)}(x) = \vec{0}$ and $\mathbf{\Pi}_i^{\star,(0)}(c_{i,x}) = \mathbf{0}$.
- Induction hypothesis:

$$\vec{V}_i^{\star,(j)}(x) = \mathbf{\Pi}_i^{\star,(j)}(c_{i,x}) \cdot \mathbf{F}_i^{\star}\hat{\vec{U}}_i(0) \ .$$

- Induction step $j \rightarrow j+1$. We obtain

$$\vec{V}_i^{\star,(j+1)}(x) = \max_{\alpha\in Act}\left\{\int_0^{c_{i,x}} \mathbf{M}_i^{\star,\alpha}(\tau)\vec{V}_i^{\star,(j)}(x+\tau)d\tau + \mathbf{D}_i^{\star}(c_{i,x}) \cdot \mathbf{F}_i^{\star}\hat{\vec{U}}_i(0)\right\} \ .$$

By induction hypothesis it follows that

$$\begin{aligned}
&\vec{V}_i^{\star,(j+1)}(x) \\
&= \max_{\alpha\in Act}\left\{\int_0^{c_{i,x}} \mathbf{M}_i^{\star,\alpha}(\tau)\vec{V}_i^{\star,(j)}(x+\tau)d\tau + \mathbf{D}_i^{\star}(c_{i,x}) \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0)\right\} \\
&= \max_{\alpha\in Act}\left\{\int_0^{c_{i,x}} \mathbf{M}_i^{\star,\alpha}(\tau) \cdot \mathbf{\Pi}_i^{\star,(j)}(c_{i,x}-\tau) \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0)d\tau + \mathbf{D}_i^{\star}(c_{i,x}) \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0)\right\} \\
&= \max_{\alpha\in Act}\left\{\left(\int_0^{c_{i,x}} \mathbf{M}_i^{\star,\alpha}(\tau)\mathbf{\Pi}_i^{\star,(j)}(c_{i,x}-\tau)d\tau + \mathbf{D}_i^{\star}(c_{i,x})\right) \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0)\right\} \\
&= \max_{\alpha\in Act}\left\{\int_0^{c_{i,x}} \mathbf{M}_i^{\star,\alpha}(\tau)\mathbf{\Pi}_i^{\star,(j)}(c_{i,x}-\tau)d\tau + \mathbf{D}_i^{\star}(c_{i,x})\right\} \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0) \\
&= \max_{\alpha\in Act}\left\{\mathbf{\Pi}_i^{\alpha,(j+1)}(c_{i,x}) \cdot \mathbf{F}_i^{\star}\hat{\vec{V}}_i(0)\right\}.
\end{aligned}$$

Clearly,

$$\mathbf{\Pi}_i^{\star}(c_{i,x}) = \lim_{j\to\infty} \mathbf{\Pi}_i^{\star,(j)}(c_{i,x})$$

and

$$\vec{V}_i^{\star}(x) = \lim_{j\to\infty} \vec{V}_i^{\star,(j)}(x).$$

Let $x = 0$ and we obtain

$$\vec{V}_i^{\star}(0) = \mathbf{\Pi}_i^{\star}(c_{i,0}) \cdot \mathbf{F}_i\hat{\vec{V}}_i(0).$$

We can also write the above relation for $x = 0$ as:

$$\begin{aligned}
\left(\frac{\vec{V}_i(0)}{\vec{V}_i'(0)}\right) &= \mathbf{\Pi}_i^{\star}(\Delta c_i)\left(\mathbf{F}_i' \middle| \mathbf{B}_i'\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\begin{array}{c|c} \mathbf{\Pi}_i(\Delta c_i) & \bar{\mathbf{\Pi}}_i^{\star}(\Delta c_i) \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\begin{array}{c|c} \mathbf{F}_i & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\begin{array}{c|c} \mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i & \bar{\mathbf{\Pi}}_i^{\star}(\Delta c_i) \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\frac{\mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i\vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}_i^{\star}(\Delta c_i)\vec{U}_0(0)}{\vec{U}_0(0)}\right).
\end{aligned}$$

As a result we can represent $\vec{V}_i(0)$ in the following matrix form

$$\vec{V}_i(0) = \mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i\vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}_i^a(\Delta c_i)\vec{U}_0(0)$$

by noting that $\mathbf{\Pi}_i$ is formed by the first k_i rows and columns of matrix $\mathbf{\Pi}_i^\star$ and $\bar{\mathbf{\Pi}}_i^\star$ is formed by the first k_i rows and the last $k_i^\star - k_i = k_0$ columns of $\mathbf{\Pi}_i^\star$. The conclusion follows form Claim 1.

For $i = m$, i.e., the last graph $\mathcal{G}_m$, the region size is infinite, therefore delay transitions do not exist. Recall that

$$\vec{U}_m(x) = \\ \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^\alpha(\tau)\vec{U}_m(x+\tau)d\tau + \vec{1}_F + \int_0^\infty \mathbf{B}_m^\alpha(\tau)d\tau \cdot \vec{U}_0(0) \right\}$$

We first prove the following claim:

Claim. For any $x \in \mathbb{R}_{\geqslant 0}$, $\vec{U}_m(x)$ is a constant vector function.

Proof of the claim. We define

$$\begin{aligned} \vec{U}_m^{(0)}(x) &= \vec{0} \\ \vec{U}_m^{(j+1)}(x) &= \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^\alpha(\tau)\vec{U}_m(x+\tau)d\tau + \vec{1}_F + \int_0^\infty \mathbf{B}_m^\alpha(\tau)d\tau \cdot \vec{U}_0(0) \right\} \end{aligned}$$

It is not difficult to see that $\vec{U}_m(x) = \lim_{j \to \infty} \vec{U}_m^{(j)}(x)$. We shall show, by induction on j, that $\vec{U}_m^{(j)}(x)$ is a constant vector function.

- Base case: $\vec{U}_m^{(0)}(x) = \vec{0}$, which is clearly constant.
- I.H.: $\vec{U}_m^{(j)}(x)$ is a constant vector function.
- Induction step: $(j \to j+1)$

$$\begin{aligned} &\vec{U}_m^{(j+1)}(x) \\ &= \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^a(\tau)\vec{U}_m^{(j)}(x+\tau)d\tau + \vec{1}_F + \int_0^\infty \mathbf{B}_m^a(\tau)d\tau \cdot \vec{U}_0(0) \right\} \\ &\overset{\text{I.H.}}{=} \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^a(\tau) \cdot \vec{U}_m^{(j)}(x)d\tau + \vec{1}_F + \int_0^\infty \mathbf{B}_m^a(\tau)d\tau \cdot \vec{U}_0(0) \right\} \\ &= \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^a(\tau)d\tau \cdot \vec{U}_m^{(j)}(x) + \vec{1}_F + \int_0^\infty \mathbf{B}_m^a(\tau)d\tau \cdot \vec{U}_0(0) \right\} \end{aligned}$$

The conclusion follows trivially.

Since $\vec{U}_m(x)$ is constant vector function, we have that

$$\vec{U}_m(x) = \\ \max_{\alpha \in Act} \left\{ \int_0^\infty \hat{\mathbf{M}}_m^\alpha(\tau)d\tau \cdot \vec{U}_m(x) + \vec{1}_F + \int_0^\infty \mathbf{B}_m^\alpha(\tau)d\tau \cdot \vec{U}_0(0) \right\}$$

More than that $\int_0^\infty \hat{\mathbf{M}}_m^\alpha(\tau)d\tau$ boils down to $\hat{\mathbf{P}}_m^\alpha$ and $\int_0^\infty \mathbf{B}_m^a(\tau)d\tau$ to $\hat{\mathbf{B}}_m^\alpha$. Also we add the vector $\vec{1}_F$ to ensure that the probability to start from a state in V_F is one. □

6 Concluding Remarks

This paper considered an extension of timed automata with exponential durations. It was shown that the region graph of such automata is a decision variant of PDPs. Two approaches were presented to determine maximal time-bounded reachability probabilities in MTA. For one-clock MTA, unbounded reachability probabilities were characterized as the solution of a linear equation whose coefficients are reachability probabilities in CTMDPs, i.e., zero-clock MTA. The paper only deals with the *locally uniform* case, i.e. the exit rate of each location in an MTA doesn't depend on the chosen action. However, all the techniques presented here can be extended to the general (non-locally uniform) case without any difficulty. Future work could be to lift our results to continuous timed games [BF09,BFK+09].

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References

[AD94] Rajeev Alur and David L. Dill. A theory of timed automata. *Theor. Comput. Sci.*, 126(2):183–235, 1994.

[BBB+07] Christel Baier, Nathalie Bertrand, Patricia Bouyer, Thomas Brihaye, and Marcus Groesser. Probabilistic and topological semantics for timed automata. In *FSTTCS*, LNCS 4855, pages 179–191, 2007.

[BBB+08] Christel Baier, Nathalie Bertrand, Patricia Bouyer, Thomas Brihaye, and Marcus Grösser. Almost-sure model checking of infinite paths in one-clock timed automata. In *LICS*, pages 217–226, 2008.

[BBBM08] Nathalie Bertrand, Patricia Bouyer, Thomas Brihaye, and Nicolas Markey. Quantitative model-checking of one-clock timed automata under probabilistic semantics. In *QEST*, pages 55–64, 2008.

[Ber95] Dimitri P. Bertsekas. *Dynamic Programming and Optimal Control.* Athena Scientific, 1995.

[BF09] Patricia Bouyer and Vojtech Forejt. Reachability in stochastic timed games. In Susanne Albers, Alberto Marchetti-Spaccamela, Yossi Matias, Sotiris E. Nikoletseas, and Wolfgang Thomas, editors, *ICALP (2)*, volume 5556 of *Lecture Notes in Computer Science*, pages 103–114. Springer, 2009.

[BFK+09] Tomás Brázdil, Vojtech Forejt, Jan Krcal, Jan Kretínský, and Antonín Kucera. Continuous-time stochastic games with time-bounded reachability. In Ravi Kannan and K. Narayan Kumar, editors, *FSTTCS*, volume 4 of *LIPIcs*, pages 61–72. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2009.

[BHKH05] Christel Baier, Holger Hermanns, Joost-Pieter Katoen, and Boudewijn R. H. M. Haverkort. Efficient computation of time-bounded reachability probabilities in uniform continuous-time Markov decision processes. *Theor. Comput. Sci.*, 345(1):2–26, 2005.

[BS10] Peter Buchholz and Ingo Schulz. Numerical analysis of continuous time Markov decision processes over finite horizons. Technical report, TU Dortmund, Germany, 2010.

[Cam97] Fabio Camilli. Approximation of integro-differential equations associated with piecewise deterministic process. *Optimal Control Applications and Methods*, 18(6):423–444, 1997.

[CD88] O. L. V. Costa and M. H. A. Davis. Approximations for optimal stopping of a piecewise-deterministic process. *Mathematics of Control, Signals, and Systems*, 1(2):123–146, 1988.

[CHKM09a] Taolue Chen, Tingting Han, Joost-Pieter Katoen, and Alexandru Mereacre. Quantitative model checking of continuous-time markov chains against timed automata specifications. In *LICS*, pages 309–318. IEEE Computer Society, 2009.

[CHKM09b] Taolue Chen, Tingting Han, Joost-Pieter Katoen, and Alexandru Mereacre. Quantitative Model Checking of Continuous-Time Markov Chains Against Timed Automata Specifications. Technical report, AIB-2009-02, RWTH Aachen University, Germany, 2009.

[Dav84] Mark H. A. Davis. Piecewise-deterministic Markov processes: A general class of non-diffusion stochastic models. *Journal of the Royal Statistical Society (B)*, 46(3):353–388, 1984.

[Dav93] Mark H. A. Davis. *Markov Models and Optimization.* Chapman and Hall, 1993.

[KNSS02] Marta Z. Kwiatkowska, Gethin Norman, Roberto Segala, and Jeremy Sproston. Automatic verification of real-time systems with discrete probability distributions. *Theor. Comput. Sci.*, 282(1):101–150, 2002.

[LL85] Suzanne M. Lenhart and Yu-Chung Liao. Integro-differential equations associated with optimal stopping time of a piecewise-deterministic process. *Stochastics*, 15(3):183–207, 1985.

[LY91] Suzanne M. Lenhart and Naoki Yamada. Perron's method for viscosity solutions associated with piecewise-deterministic processes. *Funkcialaj Ekvacioj*, 34:173–186, 1991.

[NSK09] Martin R. Neuhausser, Marielle Stoelinga, and Joost-Pieter Katoen. Delayed nondeterminism in continuous-time Markov decision processes. In Luca de Alfaro, editor, *FOSSACS*, volume 5504 of *Lecture Notes in Computer Science*, pages 364–379. Springer, 2009.

[Ver85] D. Vermes. Optimal control of piecewise-deterministic Markov process. *Stochastics*, 14:165–208, 1985.

[WJT03] S. Wang, L. S. Jennings, and K. L. Teo. Numerical solution of Hamilton-Jacobi-Bellman equations by an upwind finite volume method. *J. of Global Optimization*, 27(2-3):177–192, 2003.

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