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Quantitative Model Checking of Continuous-Time Markov Chains Against Timed Automata Specifications

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Abstract. We study the following problem: given a continuous-time Markov chain (CTMC) $\mathcal{C}$, and a linear real-time property provided as a deterministic timed automaton (DTA) $\mathcal{A}$, what is the probability of the set of paths of $\mathcal{C}$ that are accepted by $\mathcal{A}$ ($\mathcal{C}$ satisfies $\mathcal{A}$)? It is shown that this set of paths is measurable and computing its probability can be reduced to computing the reachability probability in a piecewise deterministic Markov process (PDP). The reachability probability is characterized as the least solution of a system of integral equations and is shown to be approximated by solving a system of partial differential equations. For the special case of single-clock DTA, the system of integral equations can be transformed into a system of linear equations where the coefficients are solutions of ordinary differential equations.

1 Introduction

Continuous-time Markov chains (CTMCs) are one of the most important models in performance and dependability analysis. They are exploited in a broad range of applications, and constitute the underlying semantical model of a plethora of modeling formalisms for real-time probabilistic systems such as Markovian queueing networks, stochastic Petri nets, stochastic variants of process algebras, and, more recently, calculi for system biology. CTMC model checking has been focused on the temporal logic CSL (Continuous Stochastic Logic [ASSB00,BHHK03]), a variant of timed CTL where the CTL path quantifiers are replaced by a probabilistic operator. CSL model checking proceeds — like CTL model checking — by a recursive descent over the parse tree of the formula. One of the key ingredients is that reachability probabilities for time-bounded until-formulae can be approximated arbitrarily closely by a reduction to transient analysis in CTMCs. This results in a polynomial-time algorithm that has been realized in model checkers such as PRISM and MRMC.

This paper concerns the problem of verifying CTMCs versus *linear* real-time specifications, which are based on timed automata. Concretely speaking, we explore the following problem: given a CTMC $\mathcal{C}$, and a linear real-time property provided as a *deterministic timed automaton* [AD94] (DTA) $\mathcal{A}$, what is the probability of the set of paths of $\mathcal{C}$ which are accepted by $\mathcal{A}$ ($\mathcal{C} \models \mathcal{A}$)? We consider two kinds of acceptance conditions: the reachability condition and the Muller acceptance condition. The former accepts (finite) paths which reach some final state and the latter accepts (infinite) paths that infinitely often visit some set of final states. We set off to show that this problem is well-defined in the sense that the path set is *measurable*. Computing its probability can then be reduced to computing the reachability probability in a piecewise deterministic Markov process (PDP) [Dav93], a model that is used in, e.g., stochastic control theory and financial mathematics. This result relies on a product construction of CTMC $\mathcal{C}$ and DTA $\mathcal{A}$, denoted $\mathcal{C} \otimes \mathcal{A}$, yielding *deterministic Markov timed automata* (DMTA), a variant of DTA in which, besides the usual ingredients of timed automata, like guards and clock resets, the location residence time is exponentially distributed. We show that the probability of $\mathcal{C} \models \mathcal{A}$ coincides with the reachability probability of accepting paths in $\mathcal{C} \otimes \mathcal{A}$. The underlying PDP of a DMTA is obtained by a slight adaptation of the standard region construction. The desired reachability probability is characterized as the least solution of a system of *integral equations* that is obtained from the PDP. Finally, this probability is shown to be approximated by solving a system of *partial differential equations* (PDEs). For single-clock DTA, we show that the system of integral equations can be transformed into a system of *linear equations*, where the coefficients are solutions of some *ordinary differential equations* (ODEs), which can either have an analytical solution (for small state space) or an arbitrarily closely approximated solution efficiently.

Related work is model checking of asCSL [BCH+07] and CSL$^{\text{TA}}$ [DHS09]. asCSL allows to impose a time constraint on action sequences described by regular expressions; its model-checking algorithm is based on a deterministic Rabin automaton construction. In CSL$^{\text{TA}}$, time constraints (of until modalities) are specified by *single-clock* DTA. In [DHS09], $\mathcal{C} \otimes \mathcal{A}$ is interpreted as a Markov renewal processes and model checking CSL$^{\text{TA}}$ is reduced to computing reachability probabilities in a DTMC whose transition probabilities are given by subordinate CTMCs. This technique cannot be generalized to multiple clocks. Our approach does not restrict the number of clocks and supports more specifications than CSL$^{\text{TA}}$. For the single-clock case, our approach produces the same result as [DHS09], but yields a conceptually simpler formulation whose correctness can be derived from the simplification of the system of integral equations obtained in the general case. Moreover, measurability has not been addressed in [DHS09]. Other related work [BBB+07,BBB+08,BBBM08] provides a quantitative interpretation to timed automata where delays and discrete choices are interpreted probabilistically. In this approach, delays of unbounded clocks are governed by exponential distributions like in CTMCs. Decidability results have been obtained for almost-sure properties [BBB+08] and quantitative verification [BBBM08] for (a subclass of) single-clock timed automata.

2 Preliminaries

Given a set H, let $\Pr : \mathcal{F}(H) \to [0,1]$ be a probability measure on the measurable space $(H, \mathcal{F}(H))$, where $\mathcal{F}(H)$ is a σ-algebra over H. Let $Distr(H)$ denote the set of probability measures on this measurable space.

2.1 Continuous-time Markov chains

Definition 1 (CTMC). *A (labeled)* continuous-time Markov chain *(*CTMC*) is a tuple* $\mathcal{C} = (S, \text{AP}, L, \alpha, \mathbf{P}, E)$ *where* S *is a* finite *set of* states*;* AP *is a finite set of* atomic propositions*;* $L : S \to 2^{\text{AP}}$ *is the* labeling function*;* $\alpha \in Distr(S)$ *is the* initial distribution*;* $\mathbf{P} : S \times S \to [0,1]$ *is a stochastic* transition probability matrix*; and* $E : S \to \mathbb{R}_{\geqslant 0}$ *is the* exit rate function.

The probability to exit state s as well as to take the transition $s \to s'$ in t time units is $\int_0^t E(s) \cdot e^{-E(s)\tau} d\tau$ and $\mathbf{P}(s,s') \cdot \int_0^t E(s) \cdot e^{-E(s)\tau} d\tau$, respectively. A state s is *absorbing* if $\mathbf{P}(s,s) = 1$. The *embedded discrete-time Markov chain* (DTMC) of CTMC $\mathcal{C}$ is obtained by deleting the exit rate function E, i.e., $emb(\mathcal{C}) = (S, \text{AP}, L, \alpha, \mathbf{P})$.

Definition 2 (Timed paths). *Let* $\mathcal{C}$ *be a* CTMC*.* $Paths_n^{\mathcal{C}} := S \times (\mathbb{R}_{>0} \times S)^n$ *is the set of paths of length* n *in* $\mathcal{C}$*; the set of finite paths in* $\mathcal{C}$ *is defined by* $Paths_\star^{\mathcal{C}} = \bigcup_{n \in \mathbb{N}} Paths_n^{\mathcal{C}}$ *and* $Paths_\omega^{\mathcal{C}} := (S \times \mathbb{R}_{>0})^\omega$ *is the set of infinite paths in* $\mathcal{C}$*.* $Paths^{\mathcal{C}} = Paths_\star^{\mathcal{C}} \cup Paths_\omega^{\mathcal{C}}$ *denotes the set of all paths in* $\mathcal{C}$*.*

We denote a path $\rho \in Paths^{\mathcal{C}}(s_0)$ ($\rho \in Paths(s_0)$ for short) as the sequence $\rho = s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} s_2 \cdots$ starting in state s_0 such that for $n \leqslant |\rho|$ ($|\rho|$ is the number of transitions in ρ if ρ is finite); $\rho[n] := s_n$ is the n-th state of ρ and $\rho\langle n \rangle := t_n$ is the time spent in state s_n. Let $\rho@t$ be the state occupied in ρ at time $t \in \mathbb{R}_{\geqslant 0}$, i.e. $\rho@t := \rho[n]$

where n is the smallest index such that $\sum_{i=0}^{n} \rho\langle i\rangle > t$. We assume w.l.o.g. that the time to stay in any state is strictly greater than 0.

The definition of a Borel space on paths through CTMCs follows [Var85,BHHK03]. A CTMC $\mathcal{C}$ with initial state s_0 yields a probability measure $\Pr^{\mathcal{C}}$ on paths as follows: Let $s_0, \cdots, s_k \in S$ with $\mathbf{P}(s_i, s_{i+1}) > 0$ for $0 \leqslant i < k$ and $I_0, \cdots, I_{k-1}$ nonempty intervals in $\mathbb{R}_{\geqslant 0}$, $C(s_0, I_0, \cdots, I_{k-1}, s_k)$ denotes the *cylinder set* consisting of all paths $\rho \in \mathit{Paths}(s_0)$ such that $\rho[i] = s_i$ ($i \leqslant k$), and $\rho\langle i\rangle \in I_i$ ($i < k$). $\mathcal{F}(\mathit{Paths}(s_0))$ is the smallest σ-algebra on $\mathit{Paths}(s_0)$ which contains all sets $C(s_0, I_0, \cdots, I_{k-1}, s_k)$ for all state sequences $(s_0, \cdots, s_k) \in S^{k+1}$ with $\mathbf{P}(s_i, s_{i+1}) > 0$ ($0 \leqslant i < k$) and $I_0, \cdots, I_{k-1}$ range over all sequences of nonempty intervals in $\mathbb{R}_{\geqslant 0}$. The probability measure $\Pr^{\mathcal{C}}$ on $\mathcal{F}(\mathit{Paths}(s_0))$ is the unique measure defined by induction on k by $\Pr^{\mathcal{C}}(C(s_0)) = \alpha(s_0)$ and for $k > 0$:

$$\begin{aligned}\Pr^{\mathcal{C}}\big(C(s_0, I_0, \cdots, I_{k-1}, s_k)\big) &= \Pr^{\mathcal{C}}\big(C(s_0, I_0, \cdots, I_{k-2}, s_{k-1})\big) \\ &\cdot \int_{I_{k-1}} \mathbf{P}(s_{k-1}, s_k) E(s_{k-1}) \cdot e^{-E(s_{k-1})\tau} d\tau. \end{aligned} \tag{1}$$

Example 1. An example CTMC is illustrated in Fig. 4(b) (page 13), where $\mathrm{AP} = \{a, b, c\}$ and s_0 is the initial state, i.e., $\alpha(s_0) = 1$ and $\alpha(s) = 0$ for any $s \neq s_0$. The exit rates and transition probabilities are as shown.

2.2 Deterministic timed automata

Let $\mathcal{X} = \{x_1, \ldots, x_n\}$ be a set of variables in $\mathbb{R}$. An $\mathcal{X}$-valuation is a function $\eta : \mathcal{X} \to \mathbb{R}$ assigning to each variable x a value $\eta(x)$. Let $\mathcal{V}(\mathcal{X})$ denote the set of all valuations over $\mathcal{X}$. A *constraint* over $\mathcal{X}$, denoted by g, is a subset of $\mathbb{R}^n$. Let $\mathcal{B}(\mathcal{X})$ denote the set of constraints over $\mathcal{X}$. An $\mathcal{X}$-valuation η *satisfies* constraint g, denoted as $\eta \models g$ if $(\eta(x_1), \ldots, \eta(x_n)) \in g$.

Occasionally we use a special case of *nonnegative* variables, called *clocks*. We write $\vec{0}$ for the valuation that assigns 0 to all clocks. For a subset $X \subseteq \mathcal{X}$, the reset of X, denoted $\eta[X := 0]$, is the valuation η' such that $\forall x \in X.\ \eta'(x) := 0$ and $\forall x \notin X.\ \eta'(x) := \eta(x)$. For $\delta \in \mathbb{R}_{\geqslant 0}$, $\eta + \delta$ is the valuation η'' such that $\forall x \in \mathcal{X}.\ \eta''(x) := \eta(x) + \delta$, which implies that all clocks proceed at the same speed, or equivalently, $\forall x_i \in \mathcal{X}.\ \dot{x}_i = 1$. A *clock constraint* on $\mathcal{X}$ is an expression of the form $x \bowtie c$, or $x - y \bowtie c$, or the conjunction of any clock constraints, where $x, y \in \mathcal{X}$, $\bowtie \in \{<, \leqslant, >, \geqslant\}$ and $c \in \mathbb{N}$.

Definition 3 (DTA). *A* deterministic timed automaton *is a tuple* $\mathcal{A} = (\Sigma, \mathcal{X}, Q, q_0, Q_{\mathbf{F}}, \rightarrow)$ *where*

- Σ *is a finite* alphabet*;*
- $\mathcal{X}$ *is a finite set of* clocks*;*
- Q *is a nonempty finite set of* locations*;*
- $q_0 \in Q$ *is the* initial location*;*
- $\rightarrow\ \in Q \times \Sigma \times \mathcal{B}(\mathcal{X}) \times 2^{\mathcal{X}} \times Q$ *is an* edge relation *satisfying:* $q \xrightarrow{a,g,X} q'$ *and* $q \xrightarrow{a,g',X'} q''$ *with* $g \neq g'$ *implies* $g \cap g' = \varnothing$*; and*
- $Q_{\mathbf{F}}$ *is the* Y acceptance condition*, where*
  - ▶ *if* $Y =$ reachability*, then* $Q_{\mathbf{F}} := Q_F \subseteq Q$ *is a set of* accepting locations*;*
  - ▶ *if* $Y =$ Muller*, then* $Q_{\mathbf{F}} := Q_{\mathcal{F}} \subseteq 2^Q$ *is the* acceptance family*.*

We refer to $q \xrightarrow{a,g,X} q'$ as an *edge*, where $a \in \Sigma$ is the input symbol, the *guard* g is a clock constraint on the clocks of $\mathcal{A}$, $X \subseteq \mathcal{X}$ is a set of clocks to be reset and q' is the successor location. The intuition is that the DTA $\mathcal{A}$ can move from location q to location q' when the input symbol is a and the guard g holds, while the clocks in X should be reset when entering q'. Note that we don't consider diagonal constraints like $x - y \bowtie c$ in DTA. However, it is known that this does not harm the expressiveness of a TA [BPDG98].

We will denote DTA$^\Diamond$ and DTA$^\omega$ for the DTA with reachability and Muller acceptance conditions, respectively; while with DTA we denote the general case covering both DTA$^\Diamond$ and DTA$^\omega$. As a convention, we assume each location $q \in Q_F$ in DTA$^\Diamond$ is a sink.

An (infinite) *timed path* in $\mathcal{A}$ is of the form $\theta = q_0 \xrightarrow{a_0,t_0} q_1 \xrightarrow{a_1,t_1} \cdots$, satisfying that $\eta_0 = \vec{0}$, and for all $j \geqslant 0$, it holds that $t_j > 0$, $\eta_j + t_j \models g_j$ and $\eta_{j+1} = (\eta_j + t_j)[X_j := 0]$, where η_j is the clock evaluation on *entering* q_j. Let $\mathit{inf}(\theta)$ denote the set of states $q \in Q$ such that $q = q_i$ for infinitely many $i \geqslant 0$. Furthermore, all the definitions on paths in CTMCs can be adapted.

Definition 4 (DTA accepting paths). *An infinite path θ is* accepted *by a* DTA$^\Diamond$ *if there exists some $i \geqslant 0$ such that $\theta[i] \in Q_F$; θ is accepted by a* DTA$^\omega$ *if $\mathit{inf}(\theta) \in Q_{\mathcal{F}}$.*

*Example 2 (*DTA$^\Diamond$ *and* DTA$^\omega$*).* An example DTA$^\Diamond$ is shown in Fig. 4(c) (page 13) over the alphabet $\{a, b\}$. The reachability acceptance condition is characterized by the accepting location set $Q_F = \{q_1\}$. The unique initial location is q_0 and the guards $x < 1$ and $1 < x < 2$ are disjoint on the edges labeled with a and emanating from q_0. This guarantees the determinism.

We then consider the DTA$^\omega$ in Fig. 1 over $\Sigma = \{a, b, c\}$. The unique initial location is q_0 and the Muller acceptance family is $Q_{\mathcal{F}} = \big\{\{q_0, q_2\}\big\}$. Since $Q_{\mathcal{F}}$ is a singleton, we can indicate it in the figure by the double-lined states. Any accepting path should cycle between the states q_0 and q_1 for *finitely* many times, and between states q_0 and q_2 for *infinitely* many times. The determinism is guaranteed of the similar reason.

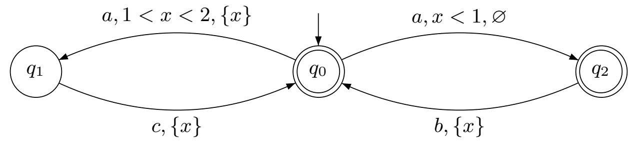


Fig. 1. DTA with Muller acceptance conditions (DTA$^\omega$)

Remark 1 (Muller not Büchi). According to *[AD94]*, the expressive power of *(deterministic) timed Muller automata (D)TMA* and *(deterministic) timed Büchi automata (D)TBA* has the following relation:

$$\mathit{TMA} = \mathit{TBA} > \mathit{DTMA} > \mathit{DTBA}.$$

Also notice that *DTMA* are closed under all Boolean operators (union, intersection and complement), while *DTBA* are *not* closed under complement. These two points justify our choice of *DTMA* (or DTA$^\omega$) instead of *DTBA*.

Remark 2 (Successor location). Due to the determinism, we can replace the transition relation $\rightarrow \in Q \times \Sigma \times \mathcal{B}(\mathcal{X}) \times 2^{\mathcal{X}} \times Q$ by a function $\mathit{succ} : Q \times \Sigma \times \mathcal{B}(\mathcal{X}) \mapsto 2^{\mathcal{X}} \times Q$. Namely, given a location q, an action a and a guard g, there is at most one successor location q'. Note that the set of reset clocks is irrelevant to the successor location. Therefore, if only the successor location is of interest, then we can thus simplify the function succ to $\widetilde{\mathit{succ}} : Q \times \Sigma \times \mathcal{B}(\mathcal{X}) \mapsto Q$, i.e., $q' = \widetilde{\mathit{succ}}(q, a, g)$.

2.3 Piecewise-Deterministic Markov Processes

The model PDP was introduced by Davis in 1984 [Dav84]. We abbreviate it as PDP instead of literally PDMP, following the convention by Davis [Dav93]. A PDP constitutes a general framework that can model virtually any stochastic system without diffusions [Dav93] and for which powerful analysis and control techniques exist [LL85,LY91,CD88]. A PDP is a stochastic process of hybrid type, i.e., the stochastic process concerns both a discrete location and a continuous variable. The class of PDPs was recognized as a very wide class holding many types of stochastic hybrid system. This makes PDP a useful model for an enormous variety of applied problems in engineering, operations research, management science and economics; examples include queueing systems, stochastic scheduling, fault detection in systems engineering, etc.

Given a set H, let $\Pr : \mathcal{F}(H) \to [0,1]$ be a probability measure on the measurable space $(H, \mathcal{F}(H))$, where $\mathcal{F}(H)$ is a σ-algebra over H. Let $Distr(H)$ denote the set of probability measures on this measurable space.

Definition 5 (PDP [Dav93]). *A* piecewise-deterministic (Markov) process *is a tuple* $\mathcal{Z} = (Z, \mathcal{X}, Inv, \phi, \Lambda, \mu)$ *with:*

- Z *is a finite set of* locations*;*
- $\mathcal{X}$ *is a finite set of* variables*;*
- $Inv : Z \to \mathcal{B}(\mathcal{X})$ *is an* invariant function*;*
- $\phi : Z \times \mathcal{V}(\mathcal{X}) \times \mathbb{R} \to \mathcal{V}(\mathcal{X})$ *is a* flow function[1]*;*
- $\Lambda : \mathbb{S} \to \mathbb{R}_{\geqslant 0}$ *is an* exit rate function*;*
- $\mu : \mathring{\mathbb{S}} \cup \partial\mathbb{S} \to Distr(\mathbb{S})$ *is the* transition probability function*, where:*

$\mathbb{S} := \{\xi := (z, \eta) \mid z \in Z, \eta \models Inv(z)\}$ *is the state space of the* PDP $\mathcal{Z}$*,* $\mathring{\mathbb{S}}$ *is the interior of* $\mathbb{S}$ *and* $\partial\mathbb{S} = \bigcup_{z \in Z}\{z\} \times \partial Inv(z)$ *is the boundary of* $\mathbb{S}$ *with* $\partial Inv(z) = \overline{Inv(z)} \setminus \mathring{Inv}(z)$ *as the boundary of* $Inv(z)$*,* $\mathring{Inv}(z)$ *the interior of* $Inv(z)$ *and* $\overline{Inv(z)}$ *the closure of* $Inv(z)$*. Functions* Λ *and* μ *satisfy the following conditions:*

- $\forall \xi \in \mathbb{S}.\ \exists \epsilon(\xi) > 0.$ *function* $t \mapsto \Lambda(\xi \oplus t)$ *is integrable on* $[0, \epsilon(\xi)[$*, where* $\xi \oplus t = \big(z, \phi(z, \eta, t)\big)$*, for* $\xi = (z, \eta)$*;*
- *Function* $\xi \mapsto \mu(\xi, A)$[2] *is measurable for any* $A \in \mathcal{F}(\mathbb{S})$*, where* $\mathcal{F}(\mathbb{S})$ *is a* σ*-algebra generated by the countable union* $\bigcup_{z \in Z}\{z\} \times A_z$ *with* A_z *being a subset of* $\mathcal{F}(Inv(z))$ *and* $\mu(\xi, \{\xi\}) = 0$*.*

We will explain the behavior of a PDP by the aid of Fig. 2. A PDP consists of a finite set of *locations* each with a *location invariant* over a set of *variables*. A *state* consists of a location and a valuation of the variables. A PDP is only allowed to stay in location z when the constraint $Inv(z)$ is satisfied. If e.g., $Inv(z)$ is $x_1^2 - 2x_2 \leqslant 1.5 \wedge x_3 > 2$, then its interior $\mathring{Inv}(z)$ is $x_1^2 - 2x_2 < 1.5 \wedge x_3 > 2$ and its closure $\overline{Inv(z)}$ is $x_1^2 - 2x_2 \leqslant 1.5 \wedge x_3 \geqslant 2$, and the boundary $\partial Inv(z)$ is $x_1^2 - 2x_2 = 1.5 \wedge x_3 = 2$. In Fig. 2, there are three locations z_0, z_1, z_2 and the gray zones are the valid valuations for respective locations. A state is a black dot. A boundary state is a white dot. When a new state $\xi = (z, \eta)$ is entered and $Inv(z)$ is valid, i.e., $\xi \in \mathbb{S}$, the PDP can (i) either *delay* to state $\xi' = (z, \eta') \in \mathbb{S} \cup \partial\mathbb{S}$ according to both the flow function ϕ and the time delay t (in this case $\xi' = \xi \oplus t$); (ii) or take a *Markovian jump* to state $\xi'' = (z'', \eta'') \in \mathbb{S}$ with probability $\mu(\xi, \{\xi''\})$. Note that the residence time of a location is exponentially distributed. When the variable valuation satisfies the boundary (i.e., $\xi \in \partial\mathbb{S}$), the PDP is *forced to take a boundary jump* and leave the current location z with probability $\mu(\xi, \{\xi''\})$ to state ξ''.

The flow function ϕ defines the time-dependent behavior in a single location, in particular, how the variable valuations change when time elapses. State $\xi \oplus t$ is the

[1] The flow function is the solution of a system of ODEs with a Lipschitz continuous vector field.

[2] $\mu(\xi, A)$ is a shorthand for $(\mu(\xi))(A)$.

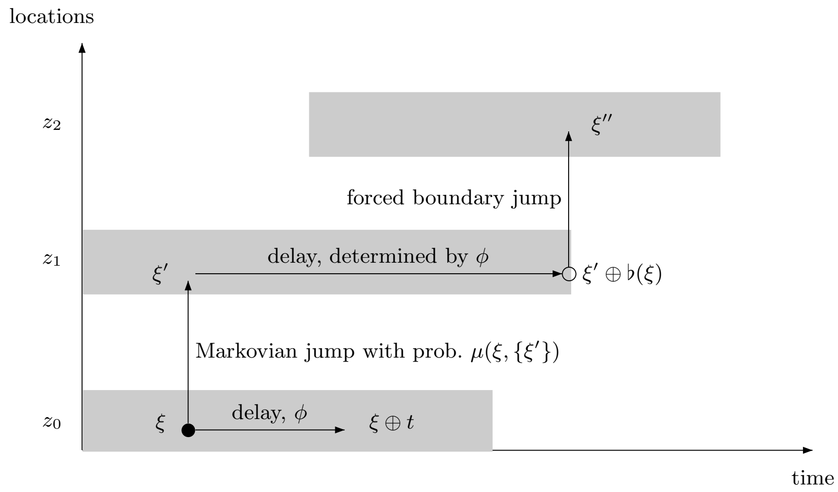


Fig. 2. The behavior of a PDP

timed successor of state ξ (on the same location) given that t time units have passed. The PDP is piecewise-deterministic because in each location (one piece) the behavior is deterministically determined by ϕ. The process is *Markovian* as the current state contains all the information to predict the future progress of the process.

The embedded *discrete-time Markov process* (DTMP) $emb(\mathcal{Z})$ of the PDP $\mathcal{Z}$ has the same state space $\mathbb{S}$ as $\mathcal{Z}$. The (one-jump) *transition probability* from a state ξ to a set $A \subseteq \mathbb{S}$ of states (on different locations as ξ), denoted $\hat{\mu}(\xi, A)$, is given by [Dav93]:

$$\hat{\mu}(\xi, A) = \int_0^{\flat(\xi)} (\mathcal{Q}\mathbf{1}_A)(\xi \oplus t)\cdot\Lambda\,(\xi \oplus t)\, e^{-\int_0^t \Lambda(\xi\oplus\tau)d\tau}\; dt \tag{2}$$

$$+\ (\mathcal{Q}\mathbf{1}_A)(\xi \oplus \flat(\xi)) \cdot e^{-\int_0^{\flat(\xi)} \Lambda(\xi\oplus\tau)d\tau}, \tag{3}$$

where $\flat(\xi) = \inf\{t > 0 \mid \xi \oplus t \in \partial\mathbb{S}\}$ is the minimal time to hit the boundary if such time exists; $\flat(\xi) = \infty$ otherwise. $(\mathcal{Q}\mathbf{1}_A)(\xi) = \int_{\mathbb{S}} \mathbf{1}_A(\xi')\mu(\xi, d\xi')$ is the accumulative (one-jump) transition probability from ξ to A and $\mathbf{1}_A(\xi)$ is the characteristic function such that $\mathbf{1}_A(\xi) = 1$ when $\xi \in A$ and $\mathbf{1}_A(\xi) = 0$ otherwise. Term (2) specifies the probability to delay to state $\xi \oplus t$ (on the same location) and take a Markovian jump from $\xi \oplus t$ to A. Note the delay t can take a value from $[0, \flat(\xi))$. Term (3) is the probability to stay in the same location for $\flat(\xi)$ time units and then it is forced to take a boundary jump from $\xi \oplus \flat(\xi)$ to A since $Inv(z)$ is invalid.

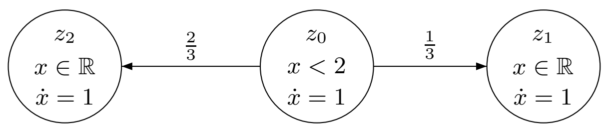


Fig. 3. An example PDP $\mathcal{Z}$

Example 3. Fig. 3 depicts a 3-location PDP $\mathcal{Z}$ with one variable x, where $Inv(z_0)$ is $x < 2$ and $Inv(z_1)$, $Inv(z_2)$ are both $x \in [0, \infty)$. Solving $\dot{x} = 1$ gives the flow function $\phi(z_i, \eta(x), t) = \eta(x) + t$ for $i = 0, 1, 2$. The state space of $\mathcal{Z}$ is $\{(z_0, \eta) \mid$

$0 < \eta(x) < 2\} \cup \{(z_1, \mathbb{R})\} \cup \{(z_2, \mathbb{R})\}$. Let exit rate $\Lambda(\xi) = 5$ for any $\xi \in \mathbb{S}$. For $\eta \models \mathit{Inv}(z_0)$, let $\mu\big((z_0, \eta), \{(z_1, \eta)\}\big) := \frac{1}{3}$, $\mu\big((z_0, \eta), \{(z_2, \eta)\}\big) := \frac{2}{3}$ and the boundary measure $\mu\big((z_0, 2), \{(z_1, 2)\}\big) := 1$. Given state $\xi_0 = (z_0, 0)$ and the set of states $A = (z_1, \mathbb{R})$, the time for ξ_0 to hit the boundary is $\flat(\xi_0) = 2$. Then $(\mathcal{Q}\mathbf{1}_A)(\xi_0 \oplus t) = \frac{1}{3}$ if $t < 2$, and $(\mathcal{Q}\mathbf{1}_A)(\xi_0 \oplus t) = 1$ if $t = 2$. In $\mathit{emb}(\mathcal{Z})$, the transition probability from state ξ_0 to A is:

$$\hat{\mu}(\xi_0, A) = \int_0^2 \frac{1}{3} \cdot 5 \cdot e^{-\int_0^t 5 \, d\tau} \, dt + 1 \cdot e^{-\int_0^2 5 \, d\tau} = \frac{1}{3} + \frac{2}{3} e^{-10}. \qquad \blacklozenge$$

3 Model checking DTA specifications

In this section, we deal with model checking linear real-time properties specified by DTA. The aim of model checking is to compute the probability of the set of paths in CTMC $\mathcal{C}$ accepted by a DTA $\mathcal{A}$. We prove that this can be reduced to computing the reachability probability in the product of $\mathcal{C}$ and $\mathcal{A}$ (Theorem 2). To simplify the notations, we assume w.l.o.g. that a CTMC has only one initial state s_0, i.e., $\alpha(s_0) = 1$, and $\alpha(s) = 0$ for $s \neq s_0$.

Definition 6 (CTMC paths accepted by a DTA). *Given a* CTMC $\mathcal{C} = (S, \mathrm{AP}, L, s_0, \mathbf{P}, E)$ *and a* DTA $\mathcal{A} = (2^{\mathrm{AP}}, \mathcal{X}, Q, q_0, Q_{\mathbf{F}}, \rightarrow)$*, a* CTMC *path* $\sigma = s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} s_2 \cdots$ *is* accepted by $\mathcal{A}$ *if the* DTA *path*

$$q_0 \xrightarrow{L(s_0), t_0} \underbrace{\widetilde{succ}\big(q_0, L(s_0), g_0\big)}_{q_1} \xrightarrow{L(s_1), t_1} \underbrace{\widetilde{succ}\big(q_1, L(s_1), g_1\big)}_{q_2} \cdots$$

is accepted by $\mathcal{A}$, where $\eta_0 = \vec{0}$, g_i is the unique guard (if it exists) such that $\eta_i + t_i \models g_i$ and $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$.

The model checking problem on CTMC $\mathcal{C}$ against DTA $\mathcal{A}$ is to compute the probability of the set of paths in $\mathcal{C}$ that can be accepted by $\mathcal{A}$. Formally, let

$$\mathit{Paths}^{\mathcal{C}}(\mathcal{A}) := \{\ \rho \in \mathit{Paths}_{\mathcal{C}} \mid \rho \text{ is accepted by DTA } \mathcal{A}\ \}.$$

Prior to computing the probability of this set, we first prove its measurability:

Theorem 1. *For any* CTMC $\mathcal{C}$ *and* DTA $\mathcal{A}$*, $\mathit{Paths}^{\mathcal{C}}(\mathcal{A})$ is measurable.*

Proof. We first deal with the case that $\mathcal{A}$ only contains *strict* inequality. Since $\mathit{Paths}^{\mathcal{C}}(\mathcal{A})$ is a set of *finite* paths, $\mathit{Paths}^{\mathcal{C}}(\mathcal{A}) = \bigcup_{n \in \mathbb{N}} \mathit{Paths}^{\mathcal{C}}_n(\mathcal{A})$, where $\mathit{Paths}^{\mathcal{C}}_n(\mathcal{A})$ is the set of accepting paths by $\mathcal{A}$ of length n. For any path $\rho := s_0 \xrightarrow{t_0} s_1 \cdots s_{n-1} \xrightarrow{t_{n-1}} s_n \in \mathit{Paths}^{\mathcal{C}}_n(\mathcal{A})$, we can associate ρ with a path $\theta := q_0 \xrightarrow{L(s_0), t_0} q_1 \cdots q_{n-1} \xrightarrow{L(s_{n-1}), t_{n-1}} q_n$ of $\mathcal{A}$ induced by the location sequence:

$$q_0 \xrightarrow{L(s_0), g_0, X_0} q_1 \cdots q_{n-1} \xrightarrow{L(s_{n-1}), g_{n-1}, X_{n-1}} q_n,$$

such that $q_n \in Q_F$ and there exist $\{\eta_i\}_{1 \leqslant i < n}$ with 1) $\eta_0 = \vec{0}$; 2) $(\eta_i + t_i) \models g_i$; and 3) $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$, where η_i is the clock valuation on entering q_i.

To prove the measurability of $\mathit{Paths}^{\mathcal{C}}_n(\mathcal{A})$, it suffices to show that for each path $\rho := s_0 \xrightarrow{t_0} \cdots \xrightarrow{t_{n-1}} s_n \in \mathit{Paths}^{\mathcal{C}}_n(\mathcal{A})$, there exists a cylinder set $C(s_0, I_0, \ldots, I_{n-1}, s_n)$ (C_ρ for short) that contains ρ and that each path in C_ρ is accepted by $\mathcal{A}$. The interval I_i is constructed according to t_i as $I_i = [t_i^-, t_i^+]$ such that

- If $t_i \in \mathbb{Q}$, then $t_i^- = t_i^+ := t_i$;

– else if $t_i \in \mathbb{R} \setminus \mathbb{Q}$, then let $t_i^-, t_i^+ \in \mathbb{Q}$ such that
  – $t_i^- \leqslant t_i \leqslant t_i^+$ and $\lfloor t_i^- \rfloor = \lfloor t_i \rfloor$ and $\lceil t_i^+ \rceil = \lceil t_i \rceil$;
  – $t_i^+ - t_i^- < \dfrac{\Delta}{2 \cdot n}$, where (with $\{\cdot\}$ denoting the fractional part) $\Delta = \min_{0 \leqslant j < n,\ x \in \mathcal{X}} \Big\{ \{\eta_j(x) + t_j\}, 1 - \{\eta_j(x) + t_j\} \ \Big|\ \{\eta_j(x) + t_j\} \neq 0 \Big\}$[3].

To show that $\rho' := s_0 \xrightarrow{t'_0} \cdots \xrightarrow{t'_{n-1}} s_n \in C_\rho$ is accepted by $\mathcal{A}$, let $\eta'_0 := \vec{0}$ and $\eta'_{i+1} := (\eta'_i + t'_i)[X_i := 0]$. We will show that $\eta'_i + t'_i \models g_i$. To this end, it suffices to observe that $\eta'_0 = \eta_0$, and for any $i > 0$ and any clock variable x,

$$\big|\eta'_i(x) - \eta_i(x)\big| \leqslant \sum_{j=0}^{i-1} \big|t'_j - t_j\big| \leqslant \sum_{j=0}^{i-1} t_j^+ - t_j^- \leqslant n \cdot (t_j^+ - t_j^-) \leqslant \frac{\Delta}{2}.$$

We claim that since DTA $\mathcal{A}$ is open, it must be the case that $\eta'_i + t'_i \models g_i$. To see this, suppose g_i is of the form $x > K$ for some integer K. We have that $|\eta'_i(x) - \eta_i(x)| \leqslant \frac{\Delta}{2}$ and $|t'_i - t_i| < \frac{\Delta}{2}$, therefore $|(\eta'_i(x) + t'_i) - (\eta_i(x) + t_i)| < \Delta$. Note that $\eta_i(x) + t_i > K$, and thus $\eta_i(x) + t_i - \{\eta_i(x) + t_i\} = \lceil \eta_i(x) + t_i \rceil \geq K$. Hence $\eta_i(x) + t_i - \Delta \geq K$ since $\Delta \leqslant \{\eta_i(x) + t_i\}$. It follows that $\eta'_i(x) + t'_i > K$. A similar argument applies to the case $x < K$ and can be extended to any constraint g_i. Thus, $\eta'_i + t'_i \models g_i$.

It follows that C_ρ is a cylinder set of $\mathcal{C}$ and each path in this cylinder set is accepted by $\mathcal{A}$, namely, $\rho \in C_\rho$ and $C_\rho \subseteq Paths_n^{\mathcal{C}}(\mathcal{A})$ with $|\rho| = n$. Together with the fact that $Paths_n^{\mathcal{C}}(\mathcal{A}) \subseteq \bigcup_{\rho \in Paths_n^{\mathcal{C}}(\mathcal{A})} C_\rho$, we have:

$$Paths_n^{\mathcal{C}}(\mathcal{A}) = \bigcup_{\rho \in Paths_n^{\mathcal{C}}(\mathcal{A})} C_\rho \quad \text{and} \quad Paths^{\mathcal{C}}(\mathcal{A}) = \bigcup_{n \in \mathbb{N}} \bigcup_{\rho \in Paths_n^{\mathcal{C}}(\mathcal{A})} C_\rho.$$

We note that each interval in the cylinder set C_ρ has rational bounds, thus C_ρ is measurable. It follows that $Paths^{\mathcal{C}}(\mathcal{A})$ is a union of *countably many* cylinder sets, and hence is measurable.

We then deal with $\mathcal{A}$ with equalities of the form $x = n$ for $n \in \mathbb{N}$. We show the measurability by induction on the number of equalities appearing in $\mathcal{A}$. We have shown the base case (DTA with only strict inequalities). Now suppose there exists a transition $\iota = q \xrightarrow{a,g,X} q'$ where g contains $x = n$. We first consider a DTA $\mathcal{A}_\iota$ obtained from $\mathcal{A}$ by deleting the transitions from q other than ι. We then consider three DTA $\bar{\mathcal{A}}_\iota$, $\mathcal{A}_\iota^>$ and $\mathcal{A}_\iota^<$ where $\bar{\mathcal{A}}_\iota$ is obtained from $\mathcal{A}_\iota$ by replacing $x = n$ by *true*; $\mathcal{A}_\iota^>$ is obtained from $\mathcal{A}_\iota$ by replacing $x = n$ by $x > n$ and $\mathcal{A}_\iota^<$ is obtained from $\mathcal{A}_\iota$ by replacing $x = n$ by $x < n$. It is not difficult to see that

$$Paths^{\mathcal{C}}(\mathcal{A}_\iota) = Paths^{\mathcal{C}}(\bar{\mathcal{A}}_\iota) \setminus (Paths^{\mathcal{C}}(\mathcal{A}_\iota^>) \cup Paths^{\mathcal{C}}(\mathcal{A}_\iota^<)).$$

Note that this holds since $\mathcal{A}$ is deterministic. By induction hypothesis, $Paths^{\mathcal{C}}(\bar{\mathcal{A}}_\iota)$, $Paths^{\mathcal{C}}(\mathcal{A}_\iota^>)$ and $Paths^{\mathcal{C}}(\mathcal{A}_\iota^<)$ are measurable. Hence $Paths^{\mathcal{C}}(\mathcal{A}_\iota)$ is measurable. Furthermore, we note that

$$Paths^{\mathcal{C}}(\mathcal{A}) = \bigcup_{\iota = q \xrightarrow{a,g,X} q'} Paths^{\mathcal{C}}(\mathcal{A}_\iota),$$

therefore $Paths^{\mathcal{C}}(\mathcal{A})$ is measurable as well.

For arbitrary $\mathcal{A}$ with time constraints of the form $x \bowtie n$ where $\bowtie \in \{\geq, \leq\}$, we consider two DTA $\mathcal{A}_=$ and $\mathcal{A}_\bowtie$. Clearly $Paths^{\mathcal{C}}(\mathcal{A}) = Paths^{\mathcal{C}}(\mathcal{A}_=) \cup Paths^{\mathcal{C}}(\mathcal{A}_{\bar{\bowtie}})$, where $\bar{\bowtie} => $ if $\bowtie = \geq$; $<$ otherwise. Clearly $Paths^{\mathcal{C}}(\mathcal{A})$ is measurable. ■

[3] Note that we are considering open timed automata. Hence for any i with $\eta_i + t_i \models g_i$, it must be the case that $\{\eta_i(x) + t_i\} \neq 0$.

3.1 Product of CTMC and DTA

As the traditional way of verifying the automata specifications, a product between the model and the automaton is built first and the (adapted) property can then be checked on the product model. Our approach is carried out in the same fashion. In this section, we focus on building the product (and some more transformations on the product) and in Section 4 and 5, the probability measure $Prob^{\mathcal{C}}(\mathcal{A})$ will be computed.

We will first exploit the product of a CTMC and a DTA, which is what we call a *deterministic Markovian timed automaton*:

Definition 7 (DMTA). *A* deterministic Markovian timed automaton *is a tuple* $\mathcal{M} = (Loc, \mathcal{X}, \ell_0, Loc_{\mathbf{F}}, E, \rightsquigarrow)$*, where*

- Loc *is a finite set of* locations*;*
- $\mathcal{X}$ *is a finite set of* clocks*;*
- $\ell_0 \in Loc$ *is the* initial location*;*
- $Loc_{\mathbf{F}}$ *is the* acceptance condition *with* $Loc_{\mathbf{F}} := Loc_F \subseteq Loc$ *the reachability condition and* $Loc_{\mathbf{F}} := Loc_{\mathcal{F}} \subseteq 2^{Loc}$ *the Muller condition;*
- $E : Loc \to \mathbb{R}_{\geqslant 0}$ *is the* exit rate function*; and*
- $\rightsquigarrow \subseteq Loc \times \mathcal{B}(\mathcal{X}) \times 2^{\mathcal{X}} \times Distr(Loc)$ *is an* edge relation *satisfying* (ℓ, g, X, ζ), $(\ell, g', X', \zeta') \in \rightsquigarrow$ *with* $g \neq g'$ *implies* $g \cap g' = \varnothing$.

The set of clocks $\mathcal{X}$ and the related concepts, e.g., clock valuation, clock constraints are defined as for DTA. We refer to $\ell \overset{g,X}{\rightsquigarrow} \zeta$ for distribution $\zeta \in Distr(Loc)$ as an *edge* and refer to $\ell \xmapsto[\zeta(\ell')]{g,X} \ell'$ as a *transition* of this edge. The intuition is that when entering location ℓ, the DMTA chooses a residence time which is governed by the exponential distribution, i.e., the probability to leave ℓ within t time units is $1 - e^{-E(\ell)t}$. When it decides to jump, at most one edge, say $\ell \overset{g,X}{\rightsquigarrow} \zeta$, due to the determinism, is enabled and the probability to jump to ℓ' is given by $\zeta(\ell')$. The DMTA is *deterministic* as it has a unique initial location and disjoint guards for all edges emanating from any location. Similar as in DTAs, DMTA$^\diamond$ and DMTA$^\omega$ are defined in an obvious way and DMTA refers to both cases.

Definition 8 (Paths in DMTAs). *Given a* DMTA $\mathcal{M}$*, an* (*infinite*) symbolic path *is of the form:*

$$\ell_0 \xmapsto[p_0]{g_0,X_0} \ell_1 \xmapsto[p_1]{g_1,X_1} \ell_2 \cdots$$

where $p_i = \zeta_i(\ell_{i+1})$ *is the transition probability of* $\ell_i \xmapsto[\zeta_i(\ell_{i+1})]{g_i,X_i} \ell_{i+1}$.

An infinite path *in* $\mathcal{M}$ (*induced from the symbolic path*) *is of the form* $\tau = \ell_0 \xrightarrow{t_0} \ell_1 \xrightarrow{t_1} \ell_2 \cdots$ *and has the property that* $\eta_0 = \vec{0}$, $(\eta_i + t_i) \models g_i$*, and* $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$ *where* $i \geqslant 0$ *and* η_i *is the clock valuation of* $\mathcal{X}$ *in* $\mathcal{M}$ *on* entering *location* ℓ_i.

The path τ *is* accepted *by a* DMTA$^\diamond$ *if there exists* $n \geqslant 0$*, such that* $\tau[n] \in Loc_F$. *It is* accepted *by* DMTA$^\omega$ *iff* $inf(\tau) \in Loc_{\mathcal{F}}$*, i.e.,* $\exists L_F \in Loc_{\mathcal{F}}$ *such that* $inf(\tau) = L_F$.

All definitions on paths in CTMC*s can be carried over to* DMTA *paths.*

DMTA *Semantics.* First we characterize the *one-jump* probability $\ell \xmapsto[\mathbf{P}(\ell,\ell')]{g,X} \ell'$ within time interval I starting at clock valuation η, denoted $p_\eta(\ell, \ell', I)$, as follows:

$$p_\eta(\ell, \ell', I) = \int_I \underbrace{E(\ell) \cdot e^{-E(\ell)\tau}}_{\text{(i) density to leave } \ell \text{ at } \tau} \cdot \underbrace{\mathbf{1}_g(\eta + \tau)}_{\text{(ii) } \eta+\tau \models g?} \cdot \underbrace{\mathbf{P}(\ell, \ell')}_{\text{(iii) probabilistic jump}} d\tau \quad (4)$$

Actually, (i) characterizes the delay τ at location ℓ which is exponentially distributed with rate $E(\ell)$; (ii) is the *characteristic function*, where $\mathbf{1}_g(\eta+\tau) = 1$, if $\eta+\tau \models g$; 0, otherwise. It compares the current valuation $\eta+\tau$ with g and rules out the paths that cannot lead to ℓ'; and (iii) indicates the probabilistic jump. Note that (i) and (iii) are features from CTMCs while (ii) is from DTA. The characteristic function is Riemann integrable as it is bounded and its support is an interval, therefore $p_\eta(\ell,\ell',I)$ is well-defined.

Based on the one-jump probability, we can now consider the probability of a set of paths. Given DMTA $\mathcal{M}$, $C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)$ is the cylinder set where $(\ell_0,\ldots,\ell_n) \in Loc^{n+1}$ and $I_i \subseteq \mathbb{R}_{\geqslant 0}$. It denotes a set of paths τ in $\mathcal{M}$ such that $\tau[i] = \ell_i$ and $\tau\langle i\rangle \in I_i$. Let $\Pr^{\mathcal{M}}_{\eta_0}(C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n))$ denote the probability of $C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)$ such that the initial clock valuation in location ℓ_0 is η_0. We define
$\Pr^{\mathcal{M}}_{\eta_0}(C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)) := \mathbb{P}^{\mathcal{M}}_0(\eta_0)$, where $\mathbb{P}^{\mathcal{M}}_i(\eta)$ for $0 \leqslant i \leqslant n$ is defined as: $\mathbb{P}^{\mathcal{M}}_n(\eta) = 1$ and for $0 \leqslant i < n$, we note that there exists a transition from ℓ_i to ℓ_{i+1} with $\ell_i \xmapsto[p_i]{g_i, X_i} \ell_{i+1}$ $(0 \leqslant i < n)$ and thus we define

$$\mathbb{P}^{\mathcal{M}}_i(\eta) = \int_{I_i} \underbrace{E(\ell_i)\cdot e^{-E(\ell_i)\tau}\cdot\mathbf{1}_{g_i}(\eta+\tau)\cdot p_i}_{(\star)} \cdot \underbrace{\mathbb{P}^{\mathcal{M}}_{i+1}(\eta')}_{(\star\star)}\, d\tau, \tag{5}$$

where $\eta' := (\eta+\tau)[X_i := 0]$. Intuitively, $\mathbb{P}^{\mathcal{M}}_i(\eta_i)$ is the probability of the suffix cylinder set starting from ℓ_i and η_i to ℓ_n. It is recursively computed by the product of the probability of taking a transition from ℓ_i to ℓ_{i+1} within time interval I_i (cf. $(\star)$ and (4)) and the probability of the suffix cylinder set from ℓ_{i+1} and η_{i+1} on (cf. $(\star\star)$). For the same reason as $p_\eta(\ell,\ell',I)$ was well-defined, $\mathbb{P}^{\mathcal{M}}_i(\eta)$ is well-defined.

*Example 4 (*DMTA$^\diamond$ *and* DMTA$^\omega$*).* The DMTA$^\diamond$ in Fig. 4(a) has initial location ℓ_0 with two edges, with guards $x<1$ and $1<x<2$. We use the small black dots to indicate distributions. Assume t time units elapsed. If $t<1$, then the upper edge is enabled and the probability to go to ℓ_1 within time t is $p_{\vec{0}}(\ell_0,\ell_1,t) = (1-e^{-r_0 t})\cdot 1$, where $E(\ell_0) = r_0$; no clock is reset. It is similar for $1<t<2$, except that x will be reset. $Loc_F = \{q_3\}$. It is obvious to see the determinism in this automaton. The DMTA$^\omega$ in Fig. 5(c) has Muller acceptance family $Loc_{\mathcal{F}} = \big\{\{\ell_1,\ell_2,\ell_3\},\{\ell_4,\ell_5,\ell_6\}\big\}$.

3.2 Product DMTAs

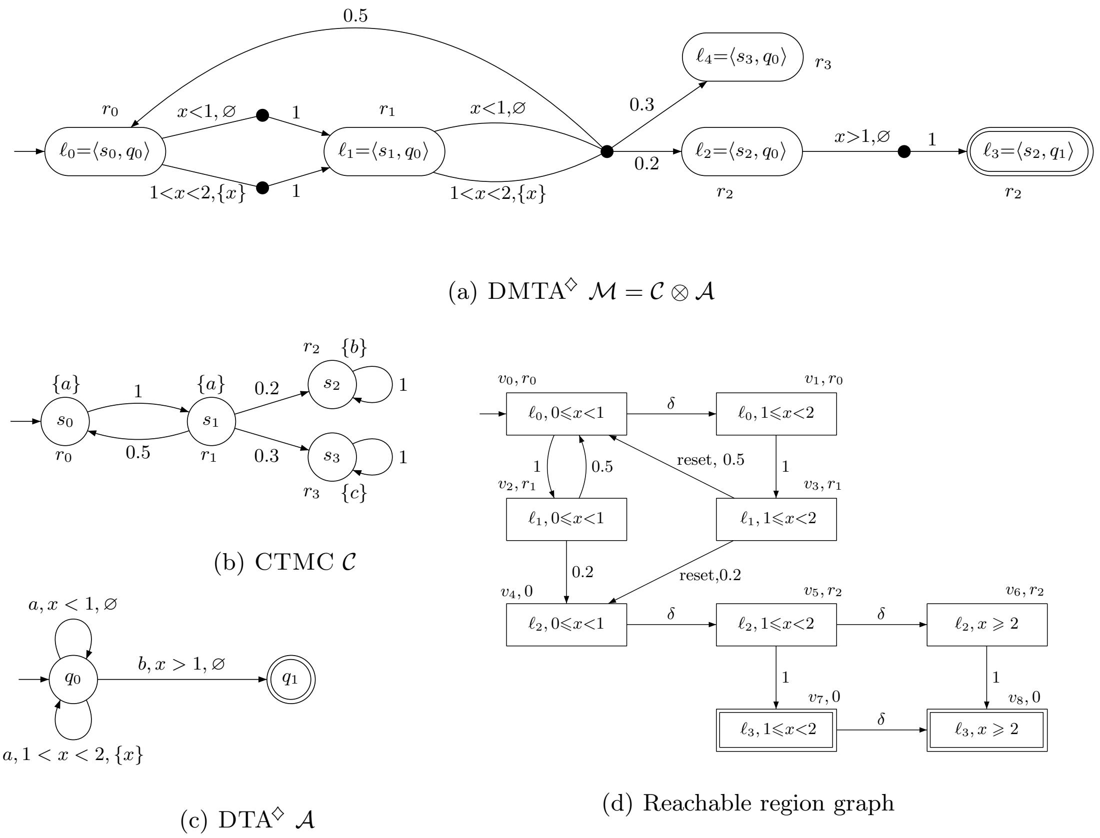
Given a CTMC $\mathcal{C}$ and a DTA $\mathcal{A}$, the product $\mathcal{C}\otimes\mathcal{A}$ is a DMTA defined by:

Definition 9 (Product of CTMC and DTA). *Let* $\mathcal{C} = (S, \mathrm{AP}, L, s_0, \mathbf{P}, E)$ *be a* CTMC *and* $\mathcal{A} = (2^{\mathrm{AP}}, \mathcal{X}, Q, q_0, Q_{\mathbf{F}}, \rightarrow)$ *be a* DTA*. We define*
$\mathcal{C}\otimes\mathcal{A} = (Loc, \mathcal{X}, \ell_0, Loc_{\mathbf{F}}, E, \rightsquigarrow)$ *as the product* DMTA*, where*

- $Loc := S \times Q$; $\quad \ell_0 := \langle s_0, q_0\rangle$; $\quad E(\langle s,q\rangle) := E(s)$;
- $Loc_{\mathbf{F}} = Loc_F := S \times Q_F$, *if* $Q_{\mathbf{F}} = Q_F$; (*reachability condition*)
  $Loc_{\mathbf{F}} = Loc_{\mathcal{F}} := \bigcup_{F\in Q_{\mathcal{F}}} S \times F$, *if* $Q_{\mathbf{F}} = Q_{\mathcal{F}}$; (*Muller condition*)
- $\rightsquigarrow$ *is defined as the smallest relation defined by the rule:*

$$\frac{\mathbf{P}(s,s') > 0 \ \wedge\ q \xrightarrow{L(s),g,X} q'}{\langle s,q\rangle \overset{g,X}{\rightsquigarrow} \zeta},\ \text{such that } \zeta(\langle s',q'\rangle) = \mathbf{P}(s,s').$$

Example 5 (Product DMTA$^\diamond$*).* Let CTMC $\mathcal{C}$ and DTA$^\diamond$ $\mathcal{A}$ be in Fig. 4(b) and 4(c), the product DMTA$^\diamond$ $\mathcal{C}\otimes\mathcal{A}$ is as in Fig. 4(a). Since $Q_F = \{q_1\}$ in $\mathcal{A}$, the set of accepting locations in DMTA$^\diamond$ is $Loc_F = \{\langle s_2, q_1\rangle\} = \{\ell_3\}$.



(a) DMTA$^\Diamond$ $\mathcal{M} = \mathcal{C} \otimes \mathcal{A}$

(b) CTMC $\mathcal{C}$

(c) DTA$^\Diamond$ $\mathcal{A}$

(d) Reachable region graph

Fig. 4. Example product DMTA$^\Diamond$ of CTMC $\mathcal{C}$ and DTA$^\Diamond$ $\mathcal{A}$

Example 6 (Product DMTA$^\omega$*).* For the CTMC $\mathcal{C}$ in Fig. 5(a) and the DTA$^\omega$ $\mathcal{A}^\omega$ in Fig. 5(b) with acceptance family $Q_\mathcal{F} = \big\{\{q_1, q_2\}, \{q_3, q_4\}\big\}$, the product DMTA$^\omega$ $\mathcal{C}\otimes\mathcal{A}^\omega$ is shown in Fig. 5(c). $Loc_\mathcal{F} = \big\{\{\langle s_i, q_1\rangle, \langle s_j, q_2\rangle\}, \{\langle s'_i, q_3\rangle, \langle s'_j, q_4\rangle\}\big\}$, for any $s_i, s'_i, s_j, s'_j \in S$, in particular, $Loc_\mathcal{F} = \big\{\{\ell_1, \ell_2, \ell_3\}, \{\ell_4, \ell_5, \ell_6\}\big\}$.

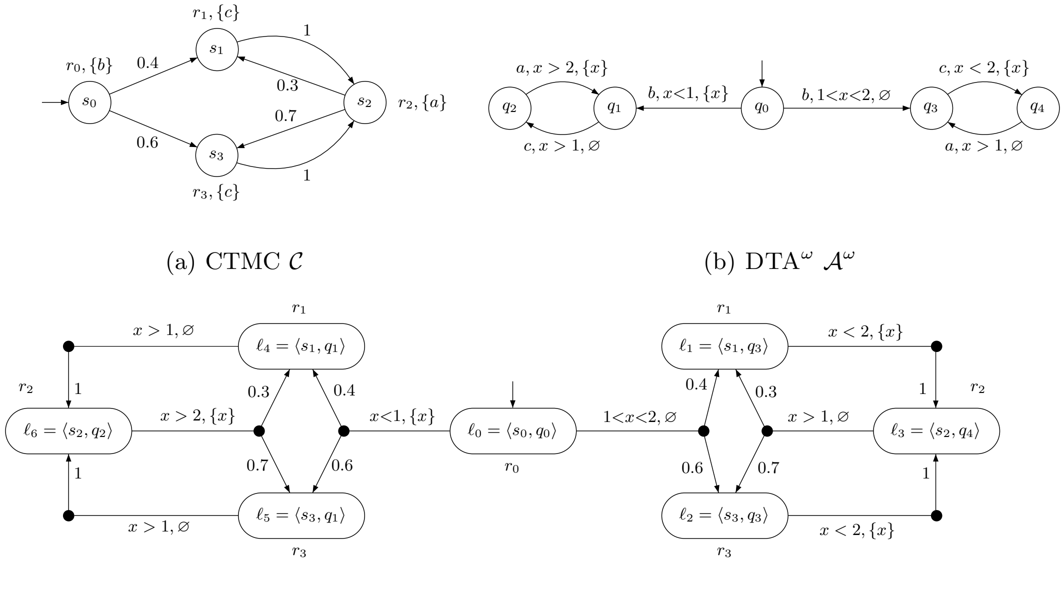
Remark 3. It is easy to see from the construction that $\mathcal{C} \otimes \mathcal{A}$ is indeed a DMTA. The determinism of the DTA $\mathcal{A}$ guarantees that the induced product is also deterministic. In $\mathcal{C} \otimes \mathcal{A}$, from each location there is at most one "action" possible, viz. $L(s)$. We can thus omit actions from the product DMTA.

For DTA$^\Diamond$ $\mathcal{A}$ with the set of accepting locations Loc_F, we denote $Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F) := \{\, \tau \in Paths^{\mathcal{C}\otimes\mathcal{A}} \mid \tau$ is accepted by $\mathcal{C}\otimes\mathcal{A}\,\}$ as the set of accepted paths in $\mathcal{C}\otimes\mathcal{A}$. Recall that $Paths^{\mathcal{C}}(\mathcal{A})$ is the set of paths in CTMC $\mathcal{C}$ that are accepted by DTA $\mathcal{A}$. For any n-ary tuple J, let $J{\downharpoonleft_i}$ denote the i-th entry in J, for $1 \leqslant i \leqslant n$. For a $(\mathcal{C}\otimes\mathcal{A})$-path $\tau = \langle s_0, q_0\rangle \xrightarrow{t_0} \langle s_1, q_1\rangle \xrightarrow{t_1} \cdots$, let $\tau{\downharpoonleft_1} := s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} \cdots$, and for any set $\mathit{\Pi}$ of $(\mathcal{C}\otimes\mathcal{A})$-paths, let $\mathit{\Pi}{\downharpoonleft_1} = \bigcup_{\tau\in\mathit{\Pi}} \tau{\downharpoonleft_1}$.

Lemma 1. *For any* CTMC $\mathcal{C}$ *and* DTA$^\Diamond$ $\mathcal{A}$*,* $Paths^{\mathcal{C}}(\mathcal{A}) = Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F){\downharpoonleft_1}$.

Proof. ($\Longrightarrow$) It is to prove that for any path $\rho \in Paths^{\mathcal{C}}(\mathcal{A})$, there exists a path $\tau \in Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)$ such that $\tau{\downharpoonleft_1} = \rho$.

We assume w.l.o.g. that $\rho = s_0 \xrightarrow{t_0} s_1 \cdots s_{n-1} \xrightarrow{t_{n-1}} s_n \in Paths^{\mathcal{C}}$ is accepted by $\mathcal{A}$, i.e., $s_n \in Q_F$ and for $0 \leqslant i < n$, $\eta_0 \models \vec{0}$ and $\eta_i + t_i \models g_i$ and $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$,



(a) CTMC $\mathcal{C}$

(b) DTA$^\omega$ $\mathcal{A}^\omega$

(c) DMTA$^\omega$ $\mathcal{C} \otimes \mathcal{A}^\omega$

Fig. 5. Example product DMTA$^\omega$ of CTMC $\mathcal{C}$ and DTA$^\omega$ $\mathcal{A}^\omega$

where η_i is the time valuation on entering state s_i. We can then construct a path $\theta \in$ *Paths*$^{\mathcal{A}}$ from ρ such that $\theta = q_0 \xrightarrow{L(s_0),t_0} q_1 \cdots q_{n-1} \xrightarrow{L(s_{n-1}),t_{n-1}} q_n$, where s_i and q_i have the same entering clock valuation. From ρ and θ, we can construct the path

$$\tau = \langle s_0, q_0 \rangle \xrightarrow{t_0} \langle s_1, q_1 \rangle \cdots \langle s_{n-1}, q_{n-1} \rangle \xrightarrow{t_{n-1}} \langle s_n, q_n \rangle,$$

where $\langle s_n, q_n \rangle \in Loc_F$. It follows from the definition of an accepting path in a DTA$^\omega$ that $\tau \in Paths^{\mathcal{C} \otimes \mathcal{A}}(\Diamond\, Loc_F)$ and $\tau\downharpoonleft_1 = \rho$.

($\Longleftarrow$) It is to prove that for any path $\tau \in Paths^{\mathcal{C} \otimes \mathcal{A}}(\Diamond\, Loc_F)$, $\tau\downharpoonleft_1 \in Paths^{\mathcal{C}}(\mathcal{A})$.

We assume w.l.o.g. that path

$$\tau = \langle s_0, q_0 \rangle \xrightarrow{t_0} \cdots \xrightarrow{t_{n-1}} \langle s_n, q_n \rangle \in Paths^{\mathcal{C} \otimes \mathcal{A}}(\Diamond\, Loc_F),$$

it holds that $\langle s_n, q_n \rangle \in Loc_F$ and for $0 \leqslant i < n$, $\eta_0 \models \vec{0}$ and $\eta_i + t_i \models g_i$ and $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$, where η_i is the time valuation on entering state $\langle s_i, q_i \rangle$. It then directly follows that $q_n \in Q_F$ and $\tau\downharpoonleft_1 \in Paths^{\mathcal{C}}(\mathcal{A})$, given η_i the entering clock valuation of state s_i. ■

The following theorem establishes the link between CTMC $\mathcal{C}$ and DMTA$^\Diamond$ $\mathcal{C} \otimes \mathcal{A}$.

Theorem 2. *For any* CTMC $\mathcal{C}$ *and* DTA$^\Diamond$ $\mathcal{A}$*,*

$$\Pr{}^{\mathcal{C}}\left(Paths^{\mathcal{C}}(\mathcal{A})\right) = \Pr{}^{\mathcal{C} \otimes \mathcal{A}}_{\vec{0}}\left(Paths^{\mathcal{C} \otimes \mathcal{A}}(\Diamond Loc_F)\right).$$

Proof. According to Theorem 1, $Paths^{\mathcal{C}}(\mathcal{A})$ can be rewritten as the combination of cylinder sets of the form $C(s_0, I_0, \ldots, I_{n-1}, s_n)$ which are all accepted by DTA $\mathcal{A}$[4]. By Lemma 1, namely by path lifting, we can establish exactly the same combination of cylinder sets $C(\ell_0, I_0, \ldots, I_{n-1}, \ell_0)$ for $Paths^{\mathcal{C} \otimes \mathcal{A}}(\Diamond Loc_F)$, where $s_i = \ell_i\downharpoonleft_1$. It then

[4] Note that this means each path in the cylinder set is accepted by $\mathcal{A}$.

suffices to show that for each cylinder set $C(s_0, I_0, \ldots, I_{n-1}, s_n)$ which is accepted by $\mathcal{A}$, $\mathrm{Pr}^{\mathcal{C}}$ and $\mathrm{Pr}^{\mathcal{C}\otimes\mathcal{A}}$ yield the same probabilities. Note that a cylinder set C is accepted by a DTA $\mathcal{A}$, if each path that C generates can be accepted by $\mathcal{A}$.

For the measure $\mathrm{Pr}^{\mathcal{C}}$, according to Eq. 1 (page 5),

$$\mathrm{Pr}^{\mathcal{C}}\big(C(s_0, I_0, \ldots, I_{n-1}, s_n)\big) = \prod_{0\leqslant i<n} \int_{I_i} \mathbf{P}(s_i, s_{i+1}) \cdot E(s_i) \cdot e^{-E(s_i)\tau} d\tau.$$

For the measure $\mathrm{Pr}^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}$, according to Section 3.1, it is given by $\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_0(\vec{0})$ where $\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_n(\eta) = 1$ for any clock valuation η and

$$\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_i(\eta_i) = \int_{I_i} \mathbf{1}_{g_i}(\eta_i + \tau_i)\cdot p_i \cdot E(\ell_i)\cdot e^{-E(\ell_i)\tau_i} \cdot \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_{i+1}(\eta_{i+1})\ d\tau_i,$$

where $\eta_{i+1} = (\eta_i + \tau_i)[X_i := 0]$ and $\mathbf{1}_{g_i}(\eta_i + \tau_i) = 1$, if $\eta_i + \tau_i \models g_i$; 0, otherwise.

We will show, by induction, that $\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_i(\eta_i)$ is a constant, i.e., is independent of η_i, if the cylinder set $C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)$ is accepted by $\mathcal{C}\otimes\mathcal{A}$. Firstly let us note that for $C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)$, there must exist some sequence of transitions

$$\ell_0 \xmapsto[p_0]{g_0, X_0} \ell_1 \cdots \ell_{n-1} \xmapsto[p_{n-1}]{g_{n-1}, X_{n-1}} \ell_n$$

with $\eta_0 = \vec{0}$ and $\forall t_i \in I_i$ with $0 \leqslant i < n$, $\eta_i + t_i \models g_i$ and $\eta_{i+1} := (\eta_i + t_i)[X_i := 0]$. Moreover, according to Def. 9, we have:

$$p_i = \mathbf{P}(s_i, s_{i+1}) \qquad \text{and} \qquad E(\ell_i) = E(s_i). \tag{6}$$

We apply a backward induction on n down to 0. The base case is trivial since $\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_n(\eta) = 1$. By I.H., $\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_{i+1}(\eta)$ is a constant. For the induction step, consider $i < n$. For any $\tau_i \in I_i$, since $\eta_i + \tau_i \models g_i$, $\mathbf{1}_{g_i}(\eta_i + \tau_i) = 1$, it follows that

$$\begin{aligned}
\mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_i(\eta_i) &= \int_{I_i} \mathbf{1}_{g_i}(\eta_i + \tau_i)\cdot p_i\cdot E(\ell_i)\cdot e^{-E(\ell_i)\tau_i} \cdot \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_{i+1}(\eta_{i+1})\ d\tau_i \\
&\overset{\text{I.H.}}{=} \int_{I_i} p_i\cdot E(\ell_i)\cdot e^{-E(\ell_i)\tau_i} d\tau_i \cdot \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_{i+1}(\eta_{i+1}) \\
&\overset{\text{Eq.(6)}}{=} \int_{I_i} \mathbf{P}(s_i, s_{i+1})\cdot E(s_i)\cdot e^{-E(s_i)\tau_i} d\tau_i \cdot \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_{i+1}(\eta_{i+1}).
\end{aligned}$$

Clearly, this is a constant. It is thus easy to see that

$$\mathrm{Pr}^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\big(C(\ell_0, I_0, \ldots, I_{n-1}, \ell_n)\big) \ := \ \mathbb{P}^{\mathcal{C}\otimes\mathcal{A}}_0(\vec{0}) = \prod_{0\leqslant i<n} \int_{I_i} \mathbf{P}(s_i, s_{i+1})\cdot E(s_i)\cdot e^{-E(s_i)\tau} d\tau,$$

which completes the proof. ■

3.3 Region Construction for DMTA

In the remainder of this section, we focus on how to compute the probability measure $\mathrm{Pr}^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\left(\mathit{Paths}^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, \mathit{Loc}_F)\right)$ in an effective way. Since the state space $\{(\ell, \eta) \mid \ell \in \mathit{Loc}, \eta \in \mathcal{V}(\mathcal{X})\}$ of $\mathcal{C}\otimes\mathcal{A}$ is uncountable, we start with adopting the standard *region construction* [AD94] to $\mathrm{DMTA}^{\Diamond}$ to discretize the state space into a finite one. As we will see in Section 3.4, this allows us to obtain a piecewise-deterministic Markov process from a $\mathrm{DMTA}^{\Diamond}$ in a natural way.

As usual, a region is a constraint. For regions $\Theta, \Theta' \in \mathcal{B}(\mathcal{X})$, Θ' is the *successor region* of Θ if for all $\eta \models \Theta$ there exists $\delta \in \mathbb{R}_{>0}$ such that $\eta + \delta \models \Theta'$ and for all $\delta' < \delta$, $\eta + \delta' \models \Theta \vee \Theta'$. A region Θ *satisfies* a guard g (denoted $\Theta \models g$) iff $\forall \eta \models \Theta.\ \eta \models g$. A *reset operation* on region Θ is defined as $\Theta[X := 0] := \{\eta[X := 0] \mid \eta \models \Theta\}$.

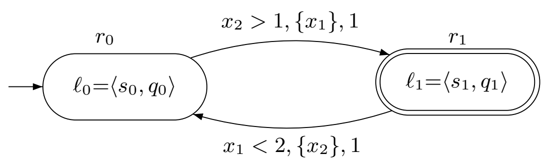
Definition 10 (Region graph of DMTA$^\diamond$). *Given* DMTA$^\diamond$ $\mathcal{M} = (Loc, \mathcal{X}, \ell_0, Loc_F, E, \rightsquigarrow)$*, the region graph is* $\mathcal{G}(\mathcal{M}) = (V, v_0, V_F, \Lambda, \hookrightarrow)$*, where*

- $V := Loc \times \mathcal{B}(\mathcal{X})$ *is a finite set of* vertices*, consisting of a location* ℓ *in* $\mathcal{M}$ *and a region* Θ*;*
- $v_0 \in V$ *is the* initial vertex *if* $(\ell_0, \vec{0}) \in v_0$*;*
- $V_F := \{v \mid v\lfloor_1 \in Loc_F\}$ *is the set of* accepting vertices*;*
- $\hookrightarrow \subseteq V \times \big(([0,1] \times 2^{\mathcal{X}}) \cup \{\delta\}\big) \times V$ *is the* transition (edge) relation*, such that:*
  - ▶ $v \overset{\delta}{\hookrightarrow} v'$ *is a* delay transition *if* $v\lfloor_1 = v'\lfloor_1$ *and* $v'\lfloor_2$ *is a successor region of* $v\lfloor_2$*;*
  - ▶ $v \overset{p,X}{\hookrightarrow} v'$ *is a* Markovian transition *if there exists some transition* $v\lfloor_1 \xmapsto[p]{g,X} v'\lfloor_1$ *in* $\mathcal{M}$ *such that* $v\lfloor_2 \models g$ *and* $v\lfloor_2[X := 0] \models v'\lfloor_2$*; and*
- $\Lambda : V \to \mathbb{R}_{\geqslant 0}$ *is the* exit rate function *where* $\Lambda(v) := E(v\lfloor_1)$ *if there exists a Markovian transition from* v*,* $\Lambda(v) := 0$ *otherwise.*

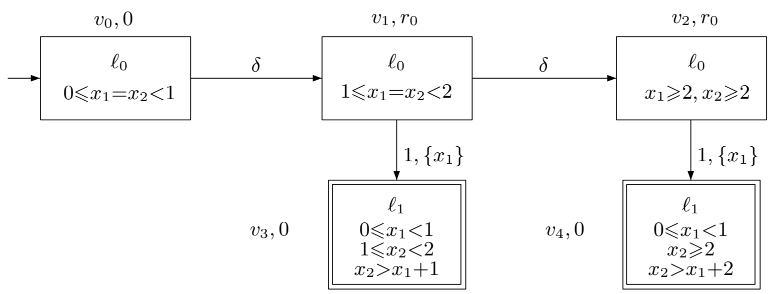
Note that in the obtained region graph, Markovian transitions emanating from any boundary region do *not* contribute to the reachability probability as the time to hit the boundary is always zero (i.e., $\flat(v, \eta) = 0$ in (8), page 18). Therefore, we can remove all the Markovian transitions emanating from boundary regions and then collapse each of them with its unique *non-boundary* (direct) successor. In the sequel, by slightly abusing the notation we still denote this *collapsed* region graph as $\mathcal{G}(\mathcal{M})$.

Remark 4 (Exit rates). The exit rate $\Lambda(v)$ is set to 0 if there is only delay transition from v. The probability to take the delay transition within time t is $e^{-\Lambda(v)t} = 1$ and the probability to take Markovian transitions is 0.

Example 7. For the DMTA$^\diamond$ $\mathcal{C}\otimes\mathcal{A}$ in Fig. 6(a), the reachable part (forward reachable from the initial vertex and backward reachable from the accepting vertices) of the collapsed region graph $\mathcal{G}(\mathcal{C}\otimes\mathcal{A})$ is shown in Fig. 6(b). The accepting vertices are sinks.



(a) DMTA$^\diamond$ $\mathcal{M} = \mathcal{C} \otimes \mathcal{A}$



(b) Reachable region graph $\mathcal{G}(\mathcal{C} \otimes \mathcal{A})$

Fig. 6. Example of a region graph

Notice that DMTA$^\diamond$ and DMTA$^\omega$ have the same locations and edge relations. The only difference is their acceptance condition. This guarantees that their obtained region graphs are the same except for the definition and interpretation of the final set V_F. We will present how V_F is derived in the region graph for DMTA$^\omega$ in Section 5.

3.4 From Region Graph to PDP

We can now define the underlying PDP of a DMTA$^\Diamond$ by using the region graph $\mathcal{G}(\mathcal{M})$. Actually, a region graph is a PDP.

Definition 11 (PDP for DMTA$^\Diamond$). *For* DMTA$^\Diamond$ $\mathcal{M} = (Loc, \mathcal{X}, \ell_0, Loc_F, E, \rightsquigarrow)$ *and region graph* $\mathcal{G}(\mathcal{M}) = (V, v_0, V_F, \Lambda, \hookrightarrow)$*, let* PDP $\mathcal{Z}(\mathcal{M}) = (V, \mathcal{X}, Inv, \phi, \Lambda, \mu)$ *where for any* $v \in V$*,*

- $Inv(v) := v\lfloor_2$ *and the state space* $\mathbb{S} := \big\{(v,\eta) \mid v \in V, \eta \in Inv(v)\big\}$*;*
- $\phi(v, \eta, t) := \eta + t$ *for* $\eta \models Inv(v)$*;*
- $\Lambda(v, \eta) := \Lambda(v)$ *is the exit rate of state* (v, η)*;*
- [boundary jump] *for each delay transition* $v \overset{\delta}{\hookrightarrow} v'$ *in* $\mathcal{G}(\mathcal{M})$ *we have* $\mu(\xi, \{\xi'\}) := 1$*, where* $\xi = (v, \eta)$*,* $\xi' = (v', \eta)$ *and* $\eta \models \partial Inv(v)$*;*
- [Markovian jump] *for each Markovian transition* $v \overset{p,X}{\hookrightarrow} v'$ *in* $\mathcal{G}(\mathcal{M})$ *we have* $\mu(\xi, \{\xi'\}) := p$*, where* $\xi = (v, \eta)$*,* $\eta \models Inv(v)$ *and* $\xi' = (v', \eta[X := 0])$*.*

From now on we write $\Lambda(v)$ instead of $\Lambda(v, \eta)$ as they coincide.

4 Model Checking DTA$^\Diamond$ Specifications

With the model and problem transformation presented in the last section, we are now ready to model check CTMC against DTA$^\Diamond$ specifications. We first consider the general case, i.e., DTA$^\Diamond$ with arbitrary number of clocks and then the special case of single clock DTA$^\Diamond$ specifications is investigated.

4.1 General DTA$^\Diamond$ Specifications

Recall that the aim of model checking is to compute the probability of the set of paths in CTMC $\mathcal{C}$ accepted by a DTA$^\Diamond$ $\mathcal{A}$. For the general case, we have proven that this is reducible to computing the reachability probability in the product $\mathcal{C} \otimes \mathcal{A}$ (Theorem 2, page 14), which can be further reduced to computing the reachability probability in a corresponding PDP (Theorem 3 below), which will be established in Section 4.1. The characterization by a system of integral equations is usually difficult to solve. Therefore we propose an approach to approximate the reachability probabilities in Section 4.1.

Characterizing Reachability Probabilities. Computing $\Pr_{\vec{0}}^{\mathcal{C}\otimes\mathcal{A}}\Big(Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)\Big)$ is now reduced to computing the (time-unbounded) reachability probability in the PDP $\mathcal{Z}(\mathcal{C} \otimes \mathcal{A})$ — basically the region graph of $\mathcal{C} \otimes \mathcal{A}$ — given the initial state $(v_0, \vec{0})$ and the set of goal states $\{(v, \eta) \mid v \in V_F, \eta \in Inv(v)\}$ $\big((V_F, \cdot)$ for short$\big)$. Reachability probabilities of untimed events in a PDP $\mathcal{Z}$ can be computed in the embedded DTMP $emb(\mathcal{Z})$. Note that the set of locations of $\mathcal{Z}$ and $emb(\mathcal{Z})$ are equal. In the sequel, let $\mathcal{D}$ denote $emb(\mathcal{Z})$.

For each vertex $v \in V$, we define recursively $Prob^{\mathcal{D}}\big((v, \eta), (V_F, \cdot)\big)$ $\big($or shortly $Prob_v^{\mathcal{D}}(\eta)\big)$ as the probability to reach the goal states $(V_F, \cdot)$ in $\mathcal{D}$ from state (v, η).

- for the delay transition $v \overset{\delta}{\hookrightarrow} v'$,

$$Prob_{v,\delta}^{\mathcal{D}}(\eta) = e^{-\Lambda(v)\flat(v,\eta)} \cdot Prob_{v'}^{\mathcal{D}}\big(\eta + \flat(v, \eta)\big). \tag{7}$$

Recall that $\flat(v, \eta)$ is the minimal time for (v, η) to hit the boundary $\partial Inv(v)$.

– for the Markovian transition $v \overset{p,X}{\hookrightarrow} v'$,

$$Prob^{\mathcal{D}}_{v,v'}(\eta) = \int_0^{\flat(v,\eta)} p{\cdot}\Lambda(v){\cdot}e^{-\Lambda(v)\tau}{\cdot}Prob^{\mathcal{D}}_{v'}\big((\eta+\tau)[X:=0]\big)\ d\tau. \tag{8}$$

Overall, for each vertex $v \in V$, we obtain:

$$Prob^{\mathcal{D}}_{v}(\eta) = \begin{cases} Prob^{\mathcal{D}}_{v,\delta}(\eta) + \sum_{v \overset{p,X}{\hookrightarrow} v'} Prob^{\mathcal{D}}_{v,v'}(\eta), & \text{if } v \notin V_F \\ 1, & \text{otherwise} \end{cases}. \tag{9}$$

Note that here the notation η is slightly abused. It represents a vector of clock variables (see Example 8). Eq. (7) and (8) are derived based on (3) and (2), respectively. In particular, the multi-step reachability probability is computed using a sequence of one-step transition probabilities.

Hence we obtain a system of *integral equations* (9). One can read (9) either in the form $f(\xi) = \int_{Dom(\xi)} K(\xi,\xi')f(d\xi')$, where K is the kernel and $Dom(\xi)$ is the domain of integration depending on the continuous state space $\mathbb{S}$; or in the operator form $f(\xi) = (\mathcal{J}f)(\xi)$, where $\mathcal{J}$ is the integration operator. Generally, (9) does *not* necessarily have a unique solution. It turns out that the reachability probability $Prob^{\mathcal{D}}_{v_0}(\vec{0})$ coincides with the least fixpoint of the operator $\mathcal{J}$ (denoted by $\text{lfp}\mathcal{J}$) i.e., $Prob^{\mathcal{D}}_{v_0}(\vec{0}) = (\text{lfp}\mathcal{J})(v_0,\vec{0})$. Formally, we have:

Theorem 3. *For any* CTMC $\mathcal{C}$ *and* DTA$^{\Diamond}$ $\mathcal{A}$, $\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\big(Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)\big)$ *is the least solution of* $Prob^{\mathcal{D}}_{v_0}(\cdot)$, *where* $\mathcal{D}$ *is the embedded* DTMP *of* $\mathcal{C}\otimes\mathcal{A}$.

Proof. We can express the set of all finite paths in $\mathcal{C}\otimes\mathcal{A}$ ending in some accepting location $\ell_n \in Loc_F$ for $n \in \mathbb{N}$ as the union over all location sequences i.e.,

$$\begin{aligned} \Pi^{\mathcal{C}\otimes\mathcal{A}} &= \bigcup_{n\in\mathbb{N}} \bigcup_{(\ell_0,\ldots,\ell_n)\in Loc^{n+1}} C(\ell_0,I_0,\ldots,I_{n-1},\ell_n) \\ &= Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F) \cup \overline{Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)}. \end{aligned}$$

where $C(\ell_0,I_0,\ldots,I_{n-1},\ell_n)$ is a cylinder set, $I_i = [0,\infty[$ and $\overline{Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)}\rfloor_1$ are the set of paths which are not accepted by the DTA $\mathcal{A}$. Notice that we can easily extend the measure $\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}$ to $\Pi^{\mathcal{C}\otimes\mathcal{A}}$ such that

$$\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\big(\Pi^{\mathcal{C}\otimes\mathcal{A}}\big) = \Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\big(Paths^{\mathcal{C}\otimes\mathcal{A}}(\Diamond\, Loc_F)\big).$$

This means that in order to prove the theorem we need to show that

$$\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}\big(\Pi^{\mathcal{C}\otimes\mathcal{A}}\big) = Prob^{\mathcal{D}}_{v_0}(\hat{\vec{0}}), \tag{10}$$

where $Prob^{\mathcal{D}}_{v_0}(\hat{\vec{0}})$ is the short form of $Prob^{\mathcal{D}}\big((v_0,\hat{\vec{0}}),(V_F,\cdot)\big)$, i.e., the reachability probability from state $(v_0,\hat{\vec{0}})$ to $(V_F,\cdot)$. Note that for better readability, we indicate clock valuations in $\mathcal{D}$ by adding a "^".

Eq. (10) is to be shown on cylinder sets. Note that each cylinder set $C(\ell_0,I_0,\ldots, I_{n-1},\ell_n) \subseteq \Pi^{\mathcal{C}\otimes\mathcal{A}}$ (C_n for short) induces a region graph $\mathcal{G}(C_n) = (V,v_0,V_F,\Lambda,\hookrightarrow)$, where its underlying PDP and embedded DTMP is $\mathcal{Z}(C_n)$ and $\mathcal{D}(C_n)$, respectively. To prove Eq. (10), it suffices to show that for each C_n,

$$\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\vec{0}}(C_n) = Prob^{\mathcal{D}(C_n)}_{v_0}(\hat{\vec{0}}),$$

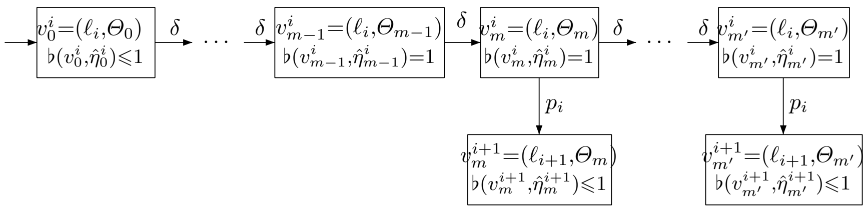
since $\Pi^{\mathcal{C}\otimes\mathcal{A}} = \bigcup_{n\in\mathbb{N}}\bigcup_{(\ell_0,\ldots,\ell_n)\in Loc^{n+1}} C_n$ and $\mathcal{D} = \bigcup_{n\in\mathbb{N}}\bigcup_{(\ell_0,\ldots,\ell_n)\in Loc^{n+1}} \mathcal{D}(C_n)$.

We will prove it by induction on the length n of the cylinder set $C_n \subseteq \Pi^{\mathcal{C}\otimes\mathcal{A}}$.

– By B.C. of $n=0$, i.e. $C_0 = C(\ell_i)$ and $\ell_i \in Loc_F$, it holds that $\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_i}(C_0) = 1$; while in the embedded DTMP $\mathcal{D}(C_0)$, since the initial vertex of $\mathcal{G}(C_0)$ is $v_0 = (\ell_i, \Theta_0)$, where $\eta_i \in \Theta_0$ and v_0 is consequently the initial location of $\mathcal{Z}(C_0)$ as well as $\mathcal{D}(C_0)$ which is accepting, $Prob^{\mathcal{D}(C_0)}_{v_0}(\hat{\eta}_i) = 1$. Note $\ell_i \in Loc$ is not necessarily the initial location ℓ_0.
– By I.H., we have that for $n = k-1$, $\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_{i+1}}(C_{k-1}) = Prob^{\mathcal{D}(C_{k-1})}_{v_{i+1}}(\hat{\eta}_{i+1})$, where $C_{k-1} = C(\ell_{i+1}, I_{i+1}, \ldots, I_{i+k-1}, \ell_{i+k})$ and $\ell_{i+k} \in Loc_F$. Note $\ell_{i+1} \in Loc$ is not necessarily the initial location ℓ_0.
– For $n = k$, let $C_k = C(\ell_i, I_i, \ell_{i+1}, I_{i+1}, \ldots, I_{i+k-1}, \ell_{i+k})$. As a result, there exists a transition $\ell_i \xmapsto[p_i]{g_i, X_i} \ell_{i+1}$ where $\eta_i + \tau_i \models g_i$ for every $\tau_i \in \,]t_1, t_2[$. $t_1, t_2 \in \mathbb{Q}_{\geqslant 0} \cup \{\infty\}$ can be obtained from g_i, such that $\tau_j \in \,]t_1, t_2[$ iff $\eta_i + \tau_j \models g_i$. According to the semantics of MTA we have

$$\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_i}(C_k) = \int_{t_1}^{t_2} p_i \cdot E(\ell_i) \cdot e^{-E(\ell_i)\tau_i} \cdot \Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_{i+1}}(C_{k-1})\, d\tau_i, \tag{11}$$

where $\eta_{i+1} = (\eta_i + \tau_i)[X_i := 0]$.



Now we deal with the inductive step for $\mathcal{D}(C_k)$. Let us assume that C_k induces the region graph $\mathcal{G}(C_k)$ whose subgraph corresponding to transition $\ell_i \xmapsto[p_i]{g_i, X_i} \ell_{i+1}$ is depicted in the figure above. For simplicity we consider that location ℓ_i induces the vertices $\{v^i_j = (\ell_i, \Theta_j) \mid 0 \leqslant j \leqslant m'\}$ and location ℓ_{i+1} induces the vertices $\{v^{i+1}_j = (\ell_{i+1}, \Theta_j) \mid m \leqslant j \leqslant m'\}$, respectively. Note that for Markovian transitions, the regions stay the same. We denote $\hat{\eta}^i_j$ (resp. $\hat{\eta}^{i+1}_j$) as the entering clock valuation on vertex v^i_j (resp. $\hat{\eta}^{i+1}_j$), for j the indices of the regions. For any $\hat{\eta} \in \bigcup_{j=0}^{m-1} \Theta_j \cup \bigcup_{j>m'} \Theta_j$, $\hat{\eta} \not\models g_i$; or more specifically,

$$t_1 = \sum_{j=0}^{m-1} \flat(v^i_j, \hat{\eta}^i_j) \qquad \text{and} \qquad t_2 = \sum_{j=0}^{m'} \flat(v^i_j, \hat{\eta}^i_j).$$

Recall that $\hat{\eta}_i$ (in the I.H.) is the clock valuation to first hit a region with ℓ_i and $\hat{\eta}_i$. Given the fact that from v^i_0 the process can only execute a delay transition before time t_1, it holds that

$$\begin{aligned}
Prob^{\mathcal{D}(C_k)}_{v^i_0}(\hat{\eta}_i) &= e^{-t_1 \Lambda(v_i)} \cdot Prob^{\mathcal{D}(C_k)}_{v^i_m}(\hat{\eta}^i_m)\\
Prob^{\mathcal{D}(C_k)}_{v^i_m}(\hat{\eta}^i_m) &= Prob^{\mathcal{D}(C_k)}_{v^i_m,\delta}(\hat{\eta}^i_m) + Prob^{\mathcal{D}(C_k)}_{v^i_m, v^{i+1}_m}(\hat{\eta}_{i+1}).
\end{aligned}$$

Therefore, we get by substitution of variables:

$$\begin{aligned}
Prob^{\mathcal{D}}_{v^i_0}(\hat{\eta}_i) &= e^{-t_1\Lambda(v_i)} \cdot Prob^{\mathcal{D}(C_k)}_{v^i_m,\delta}(\hat{\eta}^i_m) + e^{-t_1\Lambda(v_i)} \cdot Prob^{\mathcal{D}(C_k)}_{v^i_m, v^{i+1}_m}(\hat{\eta}_{i+1})\\
&= e^{-t_1\Lambda(v_i)} \cdot Prob^{\mathcal{D}(C_k)}_{v^i_m,\delta}(\hat{\eta}^i_m)\\
&+ e^{-t_1\Lambda(v_i)} \cdot \int_0^{\flat(v^i_m, \hat{\eta}^i_m)} p_i \Lambda(v_i) e^{-\Lambda(v_i)\tau} \cdot Prob^{\mathcal{D}(C_{k-1})}_{v^{i+1}_m}\big((\hat{\eta}^i_m + \tau)[X_i := 0]\big) d\tau\\
&= e^{-t_1\Lambda(v_i)} \cdot Prob^{\mathcal{D}(C_k)}_{v^i_m,\delta}(\hat{\eta}^i_m)\\
&+ \int_{t_1}^{t_1 + \flat(v^i_m, \hat{\eta}^i_m)} p_i \Lambda(v_i) e^{-\Lambda(v_i)\tau} \cdot Prob^{\mathcal{D}(C_{k-1})}_{v^{i+1}_m}\big((\hat{\eta}^i_m + \tau - t_1)[X_i := 0]\big) d\tau.
\end{aligned}$$

Evaluating each term $Prob^{\mathcal{D}(C_k)}_{v^i_m,\delta}(\hat{\eta}^i_m)$ we get the following sum of integrals:

$$
\begin{aligned}
Prob^{\mathcal{D}(C_k)}_{v^i_0}(\hat{\eta}_i) = &\sum_{j=0}^{m'-m} \int_{t_1+\sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})}^{t_1+\sum_{h=0}^{j}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})} p_i \varLambda(v_i) e^{-\varLambda(v_i)\tau} \\
&\cdot\ Prob^{\mathcal{D}(C_{k-1})}_{v^{i+1}_{m+j}}\big((\hat{\eta}^i_{m+j}+\tau-t_1-\sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h}))[X_i{:=}0]\big)d\tau.
\end{aligned}
$$

Now we define the function $F^{\mathcal{D}(C_{k-1})}(t) : [t_1, t_2] \to [0,1]$, such that when $t \in [t_1 + \sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h}), t_1 + \sum_{h=0}^{j}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})]$ for $j \leqslant m'-m$ then $F^{\mathcal{D}(C_{k-1})}(t) = Prob^{\mathcal{D}(C_{k-1})}_{v^{i+1}_{m+j}}\big((\hat{\eta}^i_{m+j} + t - t_1 - \sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h}))[X_i := 0]\big)$. Using $F^{\mathcal{D}(C_{k-1})}(t)$ we can rewrite $Prob^{\mathcal{D}(C_k)}_{v^i_0}(\hat{\eta}_i)$ to an equivalent form as:

$$
\begin{aligned}
Prob^{\mathcal{D}(C_k)}_{v^i_0}(\hat{\eta}_i) &= \sum_{j=0}^{m'-m} \int_{t_1+\sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})}^{t_1+\sum_{h=0}^{j}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})} p_i \varLambda(v_i) e^{-\varLambda(v_i)\tau} F^{\mathcal{D}(C_{k-1})}(\tau)d\tau \\
&= \int_{t_1}^{t_2} p_i \varLambda(v_i) e^{-\varLambda(v_i)\tau} F^{\mathcal{D}(C_{k-1})}(\tau)d\tau.
\end{aligned}
$$

By the I.H. we now have that for every $t \in [t_1 + \sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h}), t_1 + \sum_{h=0}^{j}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})]$ for $j \leqslant m'-m$ we have that:

$$
\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_{i+1}}(C_{k-1}) = Prob^{\mathcal{D}(C_{k-1})}_{v^{i+1}_{m+j}}\big((\hat{\eta}^i_{m+j}+t-t_1-\sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h}))[X_i{:=}0]\big) = F^{\mathcal{D}(C_{k-1})}(t),
$$

where $\eta_{i+1} = (\eta_i + t)[X_i := 0]$ and $\hat{\eta}^i_{m+j} = \hat{\eta}_i + t_1 + \sum_{h=0}^{j-1}\flat(v^i_{m+h},\hat{\eta}^i_{m+h})$. This shows that $\Pr^{\mathcal{C}\otimes\mathcal{A}}_{\eta_i}(C_k) = Prob^{\mathcal{D}(C_k)}_{v_i}(\hat{\eta}_i)$ which proves the theorem. ∎

Remark 5. Clock valuations η and η' in region $\varTheta$ may induce different reachability probabilities. The reason is that η and η' may have different periods of time to hit the boundary, thus the probability for η and η' to either delay or take a Markovian transition may differ. This is in contrast with the traditional timed automata theory as well as probabilistic timed automata *[KNSS02]*, where η and η' are not distinguished.

Example 8. For the region graph in Fig. 6(b), the system of integral equations for v_1 in location ℓ_0 is as follows for $1 \leqslant x_1 = x_2 < 2$:

$$
Prob^{\mathcal{D}}_{v_1}(x_1, x_2) = Prob^{\mathcal{D}}_{v_1,\delta}(x_1, x_2) + Prob^{\mathcal{D}}_{v_1,v_3}(x_1, x_2),
$$

where

$$
Prob^{\mathcal{D}}_{v_1,\delta}(x_1, x_2) = e^{-(2-x_1)r_0} \cdot Prob^{\mathcal{D}}_{v_2}(2, 2)
$$

and

$$
Prob^{\mathcal{D}}_{v_1,v_3}(x_1, x_2) = \int_0^{2-x_1} r_0 \cdot e^{-r_0\tau} \cdot Prob^{\mathcal{D}}_{v_3}(0, x_2 + \tau)\ d\tau
$$

where $Prob^{\mathcal{D}}_{v_3}(0, x_2 + \tau) = 1$. The integral equations for v_2 can be derived similarly.

Approximating Reachability Probabilities. Finally, we discuss how to obtain a solution of (9). The integral equations (9) are *Volterra equations of the second type* [AW95]. For a general reference on solutions to Volterra equations, cf., e.g. [Cor91]. As an alternative option to solve (9), we proceed to give a general formulation of $\Pr^{\mathcal{C}}\big(Paths^{\mathcal{C}}(\mathcal{A})\big)$ using a system of *partial differential equations* (PDEs). Let the *augmented* DTA$^\diamond$ $\mathcal{A}[t_f]$ be obtained from $\mathcal{A}$ by adding a new clock variable y which is never reset and a clock constraint $y < t_f$ on all edges entering the accepting locations in Loc_F, where t_f is a finite (and usually very large) integer. The purpose of this augmentation is to ensure that the value of all clocks reaching Loc_F is bounded. It is clear that $Paths^{\mathcal{C}}(\mathcal{A}[t_f]) \subseteq Paths^{\mathcal{C}}(\mathcal{A})$. More precisely, $Paths^{\mathcal{C}}(\mathcal{A}[t_f])$ coincides with those paths which can reach the accepting states of $\mathcal{A}$ within the time bound t_f. Note that $\lim_{t_f\to\infty} \Pr^{\mathcal{C}}(Paths^{\mathcal{C}}(\mathcal{A}[t_f])) = \Pr^{\mathcal{C}}(Paths^{\mathcal{C}}(\mathcal{A}))$. We can approximate $\Pr^{\mathcal{C}}(Paths^{\mathcal{C}}(\mathcal{A}))$ by solving the PDEs with a large t_f as follows:

Proposition 1. *Given a CTMC $\mathcal{C}$, an augmented DTA$^\diamondsuit$ $\mathcal{A}[t_f]$ and the underlying PDP $\mathcal{Z}(\mathcal{C} \otimes \mathcal{A}[t_f]) = (V, \mathcal{X}, Inv, \phi, \Lambda, \mu)$, $\Pr^{\mathcal{C}}\left(Paths^{\mathcal{C}}(\mathcal{A}[t_f])\right) = \hbar_{v_0}(0, \vec{0})$ (which is the probability to reach the final states in $\mathcal{Z}$ starting from initial state $(v_0, \vec{0}_{\mathcal{X}\cup\{y\}}$[5]$)$) is the unique solution of the following system of PDEs:*

$$\frac{\partial \hbar_v(y,\eta)}{\partial y} + \sum_{i=1}^{|\mathcal{X}|} \frac{\partial \hbar_v(y,\eta)}{\partial \eta^{(i)}} + \Lambda(v)\cdot \sum_{v \overset{p,X}{\hookrightarrow} v'} p\cdot(\hbar_{v'}(y, \eta[X := 0]) - \hbar_v(y,\eta)) = 0,$$

where $v \in V \setminus V_F$, $\eta \models Inv(v)$, $\eta^{(i)}$ is the i'th clock variable and $y \in [0, t_f)$. For every $\eta \models \partial Inv(v)$ and transition $v \overset{\delta}{\hookrightarrow} v'$, the boundary conditions take the form: $\hbar_v(y,\eta) = \hbar_{v'}(y,\eta)$. For every vertex $v \in V_F$, $\eta \models Inv(v)$ and $y \in [0, t_f)$, we have the following PDE:

$$\frac{\partial \hbar_v(y,\eta)}{\partial y} + \sum_{i=1}^{|\mathcal{X}|} \frac{\partial \hbar_v(y,\eta)}{\partial \eta^{(i)}} + 1 = 0.$$

The final boundary conditions are that for every vertex $v \in V$ and $\eta \models Inv(v) \cup \partial Inv(v)$, $\hbar_v(t_f, \eta) = 0$.

Proof. For any set of clocks $\mathcal{X}$ (n clocks) of the PDP $\mathcal{Z} = (Z, \mathcal{X}, Inv, \phi, \Lambda, \mu)$ we define a system of ODEs:

$$\frac{d\eta(y)}{dy} = \vec{1}, \eta(y_0) = \eta_0 \in \mathbb{R}^n_{\geqslant 0}, \tag{12}$$

which describe the evolution of clock values $\eta(y)$ at time y given the initial value η_0 of all clocks at time y_0. Notice that contrary to our DTA notation, Eq. (12) describes a system of ODEs where $\eta(y)$ is a vector of clock valuations at time y and $\frac{d\eta^{(i)}(y)}{dy}$ gives the timed evolution of clock $\eta^{(i)}$. Given a continuous differentiable functional $f : Z \times \mathbb{R}^n_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$, for every $z \in Z$ let:

$$\frac{df(z,\eta(y))}{dy} = \sum_{i=1}^{n} \frac{\partial f(z,\eta(y))}{\partial \eta^{(i)}} \cdot \frac{d\eta^{(i)}(y)}{dy} \overset{\text{Eq.(12)}}{=} \sum_{i=1}^{n} \frac{\partial f(z,\eta(y))}{\partial \eta^{(i)}}.$$

For notation simplicity we define the vector field from Eq. (13) as the operator Ξ which acts on functional $f(z, \eta(y))$ i.e., $\Xi f(z, \eta(y)) = \sum_{i=1}^n \frac{\partial f(z,\eta(y))}{\partial \eta^{(i)}}$. We also define the equivalent notation $\Xi f(\xi)$ for the state $\xi = (z, \eta(y))$ and any $y \in \mathbb{R}_{\geqslant 0}$.

We define the value of $\Pr^{\mathcal{C}}\left(Paths^{\mathcal{C}}(\mathcal{A})\right)$ as the expectation $\hbar(0, \xi_0)$ on PDP $\mathcal{Z}$ as follows:

$$\hbar(0,\xi_0) = \mathbb{E}\left[\int_0^{t_f} \mathbf{1}_{\mathcal{Z}}(X_\tau)d\tau \mid X_0 = \xi_0\right] = \mathbb{E}_{(0,\xi_0)}\left[\int_0^{t_f} \mathbf{1}_{\mathcal{Z}}(X_\tau)d\tau\right],$$

where the initial starting time is 0 the starting state is $\xi_0 = (z_0, \vec{0})$, X_τ is the underlying stochastic process of $\mathcal{Z}$ defined on the state space $\mathbb{S}$ and $\mathbf{1}_{\mathcal{Z}}(X_\tau) = 1$ when

[5] denoting the valuation η with $\eta(x) = 0$ for $x \in \mathcal{X} \cup \{y\}$.

$X_\tau \in \{(z, \eta(\tau)) \mid z \in V_F, \eta(\tau) \in Inv(z)\}$, $\mathbf{1}_{\mathcal{Z}}(X_\tau) = 0$, otherwise. Notice that we can also define the expectation in Eq. (13) for any starting time $y < t_f$ and state ξ as $\mathbb{E}_{(y,\xi)}\left[\int_y^{t_f} \mathbf{1}_{\mathcal{Z}}(X_\tau)d\tau\right]$.

We can obtain the expectation $\hbar(0, \xi_0)$ by following the construction in [Dav93]. For this we form the new state space $\hat{\mathbb{S}} = ([0, t_f] \times \mathbb{S}) \cup \{\Delta\}$ where Δ is the sink state and the boundary is $\partial\hat{\mathbb{S}} := ([0, t_f] \times \partial\mathbb{S}) \cup (\{t_f\} \times \mathbb{S})$. We define the following functions: $\hat{\Lambda}(y, \xi) = \Lambda(\xi)$, $\hat{\mu}((y, \xi), \{y\} \times A) = \mu(\xi, A)$ and $\hat{\mu}((t_f, \xi), \{\Delta\}) = 1$ for $y \in [0, t_f[$, $A \subseteq \mathbb{S}$ and $\xi \in \mathbb{S}$.

Given the construction we obtain an equivalent form for the expectation (13) i.e.,:

$$\hbar(0, \xi_0) = \mathbb{E}_{(0,\xi_0)}\left[\int_0^\infty \vec{\hat{1}}_{\mathcal{Z}}(\tau, X_\tau)d\tau\right], \tag{13}$$

where $\vec{\hat{1}}_{\mathcal{Z}} : \hat{\mathbb{S}} \to \{0, 1\}$, $\vec{\hat{1}}_{\mathcal{Z}}(\tau, X_\tau) = 1$ when $X_\tau \in \{(z, \eta(\tau)) \mid z \in V_F, \eta(\tau) \in Inv(z)\}$ and $\tau \in [0, t_f[$, $\vec{\hat{1}}_{\mathcal{Z}}(\tau, X_\tau) = 0$, otherwise. We also define $\vec{\hat{1}}_{\mathcal{Z}}(\Delta)$ to be zero. Notice that we introduce the sink state Δ in order to ensure that $\lim_{y\to\infty} \mathbb{E}_{(0,\xi)}\hbar(y, X_y) = 0$, which is a crucial condition in order to obtain a unique value for the expectation $\hbar(0, \xi_0)$.

For the expectation (13) [Dav93] defines the following integro-differential equations (for any $y \in [0, t_f[$):

$$\mathcal{U}\hbar(y,\xi) = \Xi\hbar(y, \xi) + \hat{\Lambda}(y, \xi) \cdot \int_{\mathbb{S}} (\hbar(y, \xi') - \hbar(y, \xi))\,\hat{\mu}((y, \xi), (y, d\xi')), \xi \in \mathbb{S} \tag{14}$$

$$\hbar(y, \xi) = \int_{\mathbb{S}} \hbar(y, \xi')\hat{\mu}((y, \xi), (y, d\xi')), \xi \in \partial\mathbb{S} \tag{15}$$

$$\mathcal{U}\hbar(y, \xi) + \vec{\hat{1}}_{\mathcal{Z}}(y, \xi) = 0, \xi \in \mathbb{S} \tag{16}$$

Equation (14) denotes the generator of the stochastic process X_y and Eq. (15) states the boundary conditions for Eq. (16). We can rewrite the integro-differential equations (14),(15) and (16) into a system of PDEs with boundary conditions given the fact that the measure $\hat{\mu}$ is not uniform. For each vertex $v \notin V_F$, $\eta \in Inv(v)$ and $y \in [0, t_f[$ of the region graph $\mathcal{G}$ we write the PDE as follows (here we define $\hbar_v(y, \eta) := \hbar(y, \xi)$ for $\xi = (v, \eta)$):

$$\frac{\partial\hbar_v(y, \eta)}{\partial y} + \sum_i \frac{\partial\hbar_v(y, \eta)}{\partial\eta^{(i)}} + \Lambda(v) \sum_{v \overset{p,X}{\hookrightarrow} v'} p \cdot (\hbar_{v'}(y, \eta[X := 0]) - \hbar_v(y, \eta)) = 0,$$

Notice that for any edge $v \overset{p,X}{\hookrightarrow} v'$ in the region graph $\mathcal{G}$, $\hat{\mu}((y, (v, \eta)), (y, (v', \eta'))) = p$. For every $\eta \in \partial Inv(v)$ and transition $v \overset{\delta}{\hookrightarrow} v'$ the boundary conditions take the form: $\hbar_v(y, \eta) = \hbar_{v'}(y, \eta)$. For every vertex $v \in V_F$, $\eta \in Inv(v)$ and $y \in [0, t_f[$ we get:

$$\frac{\partial\hbar_v(y, \eta)}{\partial y} + \sum_i \frac{\partial\hbar_v(y, \eta)}{\partial\eta^{(i)}} + 1 = 0$$

Notice that all final states are made absorbing. The final boundary conditions are that for every vertex $v \in Z$ and $\eta \in Inv(v) \cup \partial Inv(v)$, $\hbar_v(t_f, \eta) = 0$. ■

4.2 Single-Clock DTA$^\diamond$ Specifications

For single-clock DTA$^\diamond$ specifications, we can simplify the system of integral equations obtained in the previous section to a system of *linear* equations where the coefficients are a solution of a system of ODEs that can be calculated efficiently.

Given a DMTA$^\diamond$ $\mathcal{M}$, we denote the set of constants appearing in the clock constraints of $\mathcal{M}$ as $\{c_0, \ldots, c_m\}$ with $c_0 = 0$. We assume the following order: $0 = c_0 < c_1 < \cdots < c_m$.

Let $\Delta c_i = c_{i+1} - c_i$ for $0 \leqslant i < m$. Note that for one clock DMTA$^\diamondsuit$, the regions in the region graph $\mathcal{G}(\mathcal{M})$ can be represented by the following intervals: $[c_0, c_1), \ldots, [c_m, \infty)$. We partition the region graph $\mathcal{G}(\mathcal{M}) = (V, v_0, V_F, \Lambda, \hookrightarrow)$, or $\mathcal{G}$ for short, into a set of subgraphs $\mathcal{G}_i = (V_i, V_{Fi}, \Lambda_i, \{M_i, F_i, B_i\})$, where $0 \leqslant i \leqslant m$ and $\Lambda_i(v) = \Lambda(v)$, if $v \in V_i$, 0 otherwise. These subgraphs are obtained by partitioning V, V_F and $\hookrightarrow$ as follows:

- $V = \bigcup_{0 \leqslant i \leqslant m} \{V_i\}$, where $V_i = \{(\ell, \Theta) \in V \mid \Theta \subseteq [c_i, c_{i+1})\}$;
- $V_F = \bigcup_{0 \leqslant i \leqslant m} \{V_{Fi}\}$, where $v \in V_{Fi}$ iff $v \in V_i \cap V_F$;
- $\hookrightarrow = \bigcup_{0 \leqslant i \leqslant m} \{M_i \cup F_i \cup B_i\}$, where
  - M_i is the set of *Markovian transitions* (*without reset*) between vertices inside $\mathcal{G}_i$;
  - F_i is the set of *delay transitions* from the vertices in $\mathcal{G}_i$ to that in $\mathcal{G}_{i+1}$ (Forward);
  - B_i is the set of *Markovian transitions* (*with reset*) from $\mathcal{G}_i$ to $\mathcal{G}_0$ (Backward).

  It is easy to see that M_i, F_i, and B_i are pairwise disjoint.

Since the initial vertex of $\mathcal{G}_0$ is v_0 and the initial vertices of $\mathcal{G}_i$ for $0 < i \leqslant m$ are implicitly given by F_{i-1}, we omit them in the definition.

Example 9. Given the region graph in Fig. 7, the vertices are partitioned as indicated by the ovals. The M_i edges are unlabeled while the F_i and B_i edges are labeled with δ and "reset", respectively. The V_F vertices (double circles) may appear in any $\mathcal{G}_i$. Actually, if $v = (\ell, [c_i, c_{i+1})) \in V_F$, then $v' = (\ell, [c_j, c_{j+1})) \in V_F$ for $i < j \leqslant m$. This is true because $V_F = \{(\ell, \mathsf{true}) \mid \ell \in Loc_F\}$. It implies that for each final vertex not in the last region, there is a delay transition from it to the next region, see e.g. the final vertex in $\mathcal{G}_{i+1}$ in Fig. 7. The exit rate functions and the probabilities on Markovian edges are omitted in the graph.

Given a subgraph $\mathcal{G}_i$ ($0 \leqslant i \leqslant m$) of $\mathcal{G}$ with k_i states, let the probability vector $\vec{U}_i(x) = [u_i^1(x), \ldots, u_i^{k_i}(x)]^\top \in \mathbb{R}^{k_i \times 1}$ where $u_i^j(x)$ is the probability to go from vertex $v_i^j \in V_i$ to some vertex in V_F (in $\mathcal{G}$) at time x. Starting from (7)-(9), we provide a set of integral equations for $\vec{U}_i(x)$ which we later on reduce to a system of linear equations. Distinguish two cases:

Case $0 \leqslant i < m$**:** $\vec{U}_i(x)$ is given by:

$$\vec{U}_i(x) = \int_0^{\Delta c_i - x} \mathbf{M}_i(\tau) \vec{U}_i(x+\tau) d\tau + \int_0^{\Delta c_i - x} \mathbf{B}_i(\tau) d\tau \cdot \vec{U}_0(0) + \mathbf{D}_i(\Delta c_i - x) \cdot \mathbf{F}_i \vec{U}_{i+1}(0), \tag{17}$$

where $x \in [0, \Delta c_i]$ and

- $\mathbf{D}_i(x) \in \mathbb{R}^{k_i \times k_i}$ is the delay probability matrix, where for any $0 \leqslant j \leqslant k_i$, $\mathbf{D}_i(x)[j,j] = e^{-E(v_i^j)x}$ (the off-diagonal elements are zero);
- $\mathbf{M}_i(x) = \mathbf{D}_i(x) \cdot \mathbf{E}_i \cdot \mathbf{P}_i \in \mathbb{R}^{k_i \times k_i}$ is the probability density matrix for the Markovian transitions inside $\mathcal{G}_i$, where $\mathbf{P}_i$ and $\mathbf{E}_i$ are the transition probability matrix and exit rate matrix for vertices inside $\mathcal{G}_i$, respectively;
- $\mathbf{B}_i(x) \in \mathbb{R}^{k_i \times k_0}$ is the probability density matrix for the reset edges B_i, where $\mathbf{B}_i(x)[j, j']$ indicates the probability density function to take the Markovian jump with reset from the j-th vertex in $\mathcal{G}_i$ to the j'-th vertex in $\mathcal{G}_0$; and
- $\mathbf{F}_i \in \mathbb{R}^{k_i \times k_{i+1}}$ is the incidence matrix for delay edges F_i. More specifically, $\mathbf{F}_i[j, j'] = 1$ indicates that there is a delay transition from the j-th vertex in $\mathcal{G}_i$ to the j'-th vertex in $\mathcal{G}_{i+1}$; 0 otherwise.

Let us explain these equations. The third summand of (17) is obtained from (7) where $\mathbf{D}_i(\Delta c_i - x)$ indicates the probability to delay until the "end" of region i, and $\mathbf{F}_i \vec{U}_{i+1}(0)$ denotes the probability to continue in $\mathcal{G}_{i+1}$ (at relative time 0). Similarly, the first and second summands are obtained from (8); the former reflects the case where clock x is not reset, while the latter considers the reset of x (thus, implying a return to $\mathcal{G}_0$).

Case $i = m$**:** $\vec{U}_m(x)$ is simplified as follows:

$$\vec{U}_m(x) = \int_0^\infty \hat{\mathbf{M}}_m(\tau)\vec{U}_m(x+\tau)d\tau + \vec{1}_F + \int_0^\infty \mathbf{B}_m(\tau)d\tau \cdot \vec{U}_0(0) \tag{18}$$

where $\hat{\mathbf{M}}_m(\tau)[v,\cdot] = \mathbf{M}_m(\tau)[v,\cdot]$ for $v \notin V_F$, 0 otherwise. $\vec{1}_F$ is a vector such that $\vec{1}_F[v] = 1$ if $v \in V_F$, 0 otherwise. We note that $\vec{1}_F$ stems from the second clause of (9), and $\hat{\mathbf{M}}_m$ is obtained by setting the corresponding elements of $\mathbf{M}_m$ to 0. Also note that as the last subgraph $\mathcal{G}_m$ involves infinite regions, it has no delay transitions.

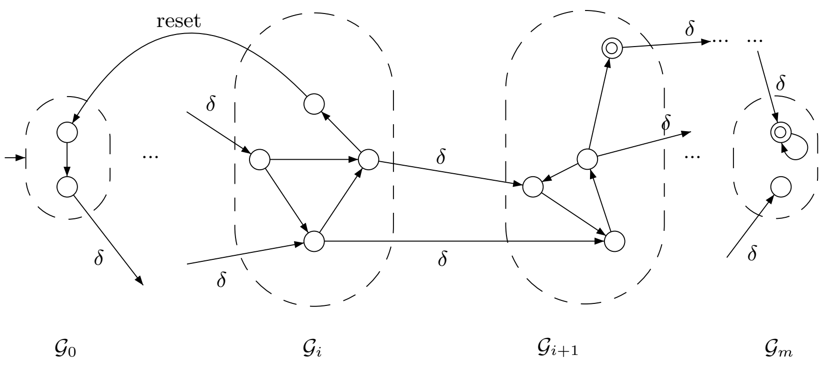


Fig. 7. Partitioning the region graph

Before solving the system of integral equations (17)-(18), we first make the following observations:

(i) Due to the fact that inside $\mathcal{G}_i$ there are only Markovian jumps with neither resets nor delay transitions, $\mathcal{G}_i$ with (V_i, Λ_i, M_i) forms a CTMC $\mathcal{C}_i$, say. For each $\mathcal{G}_i$ we define an *augmented* CTMC $\mathcal{C}_i^a$ with state space $V_i \cup V_0$, such that all V_0-vertices are made absorbing in $\mathcal{C}_i^a$. The edges connecting V_i to V_0 are kept and all the edges inside $\mathcal{C}_0$ are removed. The augmented CTMC is used to calculate the probability to start from a vertex in $\mathcal{G}_i$ and take a reset edge within a certain period of time.

(ii) Given any CTMC $\mathcal{C}$ with k states and rate matrix $\mathbf{P} \cdot \mathbf{E}$, the matrix $\mathbf{\Pi}(x)$ is given by:

$$\mathbf{\Pi}(x) = \int_0^x \mathbf{M}(\tau)\mathbf{\Pi}(x-\tau)d\tau + \mathbf{D}(x). \tag{19}$$

Intuitively, $\mathbf{\Pi}(t)[j,j']$ indicates the probability to start from vertex j and reach j' at time t.

The following proposition states the close relationship between $\mathbf{\Pi}(x)$ and the transient probability vector:

Proposition 2. *Given a* CTMC $\mathcal{C}$ *with initial distribution* α*, rate matrix* $\mathbf{P}{\cdot}\mathbf{E}$ *and* $\mathbf{\Pi}(t)$*,* $\vec{\wp}(t)$ *satisfies the following two equations:*

$$\vec{\wp}(t) = \alpha \cdot \mathbf{\Pi}(t), \tag{20}$$

$$\frac{d\vec{\wp}(t)}{dt} = \vec{\wp}(t) \cdot \mathbf{Q}, \tag{21}$$

where $\mathbf{Q} = \mathbf{P}{\cdot}\mathbf{E} - \mathbf{E}$ *is the infinitesimal generator.*

Proof. The transition probability matrix $\mathbf{\Pi}(t)$ for a CTMC $\mathcal{C}$ with state space S is denoted by the following system of integral equations:

$$\mathbf{\Pi}(t) = \int_0^t \mathbf{M}(\tau)\mathbf{\Pi}(t-\tau)d\tau + \mathbf{D}(t), \tag{22}$$

where $\mathbf{M}(\tau) = \mathbf{P}{\cdot}\mathbf{E}{\cdot}\mathbf{D}(\tau)$. Now we define for the CTMC $\mathcal{C}$ a stochastic process $X(t)$. The probability $\Pr(X(t+\mathit{\Delta} t) = s_j)$ to be in state s_j at time $t + \mathit{\Delta} t$ can be defined as:

$$\Pr(X(t+\mathit{\Delta} t) = s_j) = \sum_{s_i \in S} \Pr(X(t) = s_i) \cdot \Pr(X(t+\mathit{\Delta} t) = s_j | X(t) = s_i)$$

We can define $\Pr(X(t+\mathit{\Delta} t) = s_j)$ in the vector form as follows:

$$\vec{\wp}(t+\mathit{\Delta} t) = \vec{\wp}(t)\mathbf{\Phi}(t, t+\mathit{\Delta} t),$$

where $\vec{\wp}(t) = [\Pr(X(t) = s_1), \ldots, \Pr(X(t) = s_n)]$ and $\mathbf{\Phi}(t, t+\mathit{\Delta} t)[i,j] = \Pr(X(t+\mathit{\Delta} t) = s_j | X(t) = s_i)$.

As the stochastic process $X(t)$ is time-homogeneous we have that

$$\Pr(X(t+\mathit{\Delta} t) = s_j | X(t) = s_i) = \Pr(X(\mathit{\Delta} t) = s_j | X(0) = s_i),$$

which means that $\mathbf{\Phi}(t, t+\mathit{\Delta} t) = \mathbf{\Phi}(0, \mathit{\Delta} t)$. As $\Pr(X(\mathit{\Delta} t) = s_j | X(0) = s_i)$ denotes the transition probability to go from state s_i to state s_j at time $\mathit{\Delta} t$ we have that $\mathbf{\Phi}(0, \mathit{\Delta} t) = \mathbf{\Pi}(\mathit{\Delta} t)$, which results in the equation:

$$\vec{\wp}(t+\mathit{\Delta} t) = \vec{\wp}(t)\mathbf{\Pi}(\mathit{\Delta} t). \tag{23}$$

Now we transform Eq. (23) as follows:

$$\begin{aligned}
& \vec{\wp}(t+\mathit{\Delta} t) = \vec{\wp}(t)\mathbf{\Pi}(\mathit{\Delta} t) \\
\Longrightarrow\ & \vec{\wp}(t+\mathit{\Delta} t) - \vec{\wp}(t) = \vec{\wp}(t)\mathbf{\Pi}(\mathit{\Delta} t) - \vec{\wp}(t) \\
\Longrightarrow\ & \vec{\wp}(t+\mathit{\Delta} t) - \vec{\wp}(t) = \vec{\wp}(t)(\mathbf{\Pi}(\mathit{\Delta} t) - \mathbf{I}) \\
\Longrightarrow\ & \frac{d\vec{\wp}(t)}{dt} = \lim_{\mathit{\Delta} t \to 0} \frac{\vec{\wp}(t+\mathit{\Delta} t) - \vec{\wp}(t)}{\mathit{\Delta} t} = \vec{\wp}(t) \lim_{\mathit{\Delta} t \to 0} \frac{\mathbf{\Pi}(\mathit{\Delta} t) - \mathbf{I}}{\mathit{\Delta} t}.
\end{aligned}$$

Now it is to compute $\lim_{\mathit{\Delta} t \to 0} \frac{\mathbf{\Pi}(\mathit{\Delta} t) - \mathbf{I}}{\mathit{\Delta} t}$. For this we rewrite the right hand limit as:

$$\lim_{\mathit{\Delta} t \to 0} \frac{1}{\mathit{\Delta} t} \int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau + \lim_{\mathit{\Delta} t \to 0} \frac{1}{\mathit{\Delta} t} \left(\mathbf{D}(\mathit{\Delta} t) - \mathbf{I}\right).$$

The $\lim_{\mathit{\Delta} t \to 0} \frac{1}{\mathit{\Delta} t} \int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau$ is of the type $\frac{0}{0}$, which means we have to use l'Hospital rule:

$$\begin{gathered}
\frac{d(\mathit{\Delta} t)}{d\mathit{\Delta} t} = 1, \\
\frac{d}{d\mathit{\Delta} t}\left(\int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau\right) = \mathbf{M}(\mathit{\Delta} t)\mathbf{\Pi}(0) + \int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\frac{\partial}{\partial \mathit{\Delta} t}\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau.
\end{gathered}$$

Notice that $\mathbf{\Pi}(0) = \mathbf{I}$ and we obtain:

$$\begin{aligned}
& \lim_{\mathit{\Delta} t \to 0} \frac{1}{\mathit{\Delta} t} \int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau \\
= & \lim_{\mathit{\Delta} t \to 0} \left(\mathbf{M}(\mathit{\Delta} t)\mathbf{\Pi}(0) + \int_0^{\mathit{\Delta} t} \mathbf{M}(\tau)\frac{\partial}{\partial \mathit{\Delta} t}\mathbf{\Pi}(\mathit{\Delta} t - \tau)d\tau\right) = \mathbf{M}(0)\mathbf{\Pi}(0) = \mathbf{P}{\cdot}\mathbf{E}.
\end{aligned}$$

The $\lim_{\Delta t \to 0} \frac{1}{\Delta t} (\mathbf{D}(\Delta t) - \mathbf{I})$ is of the type $\frac{0}{0}$, which means the use of l'Hospital rule:

$$\frac{d(\Delta t)}{d\Delta t} = 1$$
$$\frac{d}{d\Delta t} (\mathbf{D}(\Delta t) - \mathbf{I}) = -\mathbf{E}\mathbf{D}(\Delta t)$$

Therefore, we obtain $\lim_{\Delta t \to 0} \frac{1}{\Delta t} (\mathbf{D}(\Delta t) - \mathbf{I}) = -\mathbf{E}$ and

$$\lim_{\Delta t \to 0} \frac{\mathbf{\Pi}(\Delta t) - \mathbf{I}}{\Delta t} = \mathbf{P} \cdot \mathbf{E} - \mathbf{E} = \mathbf{Q},$$

where $\mathbf{Q}$ is the infinitesimal generator of the CTMC $\mathcal{C}$. As a result we obtain:

$$\frac{d\vec{\wp}(t)}{dt} = \vec{\wp}(t) \lim_{\Delta t \to 0} \frac{\mathbf{\Pi}(\Delta t) - \mathbf{I}}{\Delta t} = \vec{\wp}(t)\mathbf{Q}.$$

Combining with Eq. (23) we get:

$$\vec{\wp}(t) = \alpha \cdot \mathbf{\Pi}(t),$$
$$\frac{d\vec{\wp}(t)}{dt} = \vec{\wp}(t) \cdot \mathbf{Q}. \qquad \blacksquare$$

$\vec{\wp}(t)$ is the *transient probability vector* with $\wp_s(t)$ indicating the probability to be in state s at time t given the initial probability distribution α. Eq. (21) is the celebrated forward Chapman-Kolmogorov equations. According to this proposition, solving the integral equation $\mathbf{\Pi}(t)$ boils down to selecting the appropriate initial distribution vector α and solving the system of ODEs (21), which can be done very efficiently using *uniformization*.

Prior to exposing how to solve the system of integral equations by solving a system of *linear* equations, we define $\bar{\mathbf{\Pi}}_i^a \in \mathbb{R}^{k_i \times k_0}$ for an augmented CTMC $\mathcal{C}_i^a$ to be part of $\mathbf{\Pi}_i^a$, where $\bar{\mathbf{\Pi}}_i^a$ only keeps the probabilities starting from V_i and ending in V_0. Actually,

$$\mathbf{\Pi}_i^a(x) = \left(\begin{array}{c|c} \mathbf{\Pi}_i(x) & \bar{\mathbf{\Pi}}_i^a(x) \\ \mathbf{0} & \mathbf{I} \end{array} \right),$$

where $\mathbf{0} \in \mathbb{R}^{k_0 \times k_i}$ is the zero matrix and $\mathbf{I} \in \mathbb{R}^{k_0 \times k_0}$ is the identity matrix.

Theorem 4. *For subgraph $\mathcal{G}_i$ of $\mathcal{G}$ with k_i states, it holds for $0 \leqslant i < m$ that:*

$$\vec{U}_i(0) = \mathbf{\Pi}_i(\Delta c_i) \cdot \mathbf{F}_i \vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}_i^a(\Delta c_i) \cdot \vec{U}_0(0), \tag{24}$$

where $\mathbf{\Pi}_i(\Delta c_i)$ and $\bar{\mathbf{\Pi}}_i^a(\Delta c_i)$ are for CTMC $\mathcal{C}_i$ *and the augmented* CTMC $\mathcal{C}_i^a$*, respectively. For case $i = m$,*

$$\vec{U}_m(0) = \hat{\mathbf{P}}_i \cdot \vec{U}_m(0) + \vec{1}_F + \hat{\mathbf{B}}_m \cdot \vec{U}_0(0), \tag{25}$$

where $\hat{\mathbf{P}}_i(v, v') = \mathbf{P}_i(v, v')$ if $v \notin V_F$; 0 otherwise and $\hat{\mathbf{B}}_m = \int_0^\infty \mathbf{B}_m(\tau) d\tau$.

Proof. We first deal with the case $i < m$. If in $\mathcal{G}_i$, there exists some backward edge, namely, for some j, j', $\mathbf{B}_i(x)[j, j'] \neq 0$, then we shall consider the *augmented* CTMC $\mathcal{C}_i^a$ with $k_i^a = k_i + k_0$ states. In view of this, the augmented integral equation $\vec{U}_i^a(x)$ is defined as:

$$\vec{U}_i^a(x) = \int_0^{\Delta c_i - x} \mathbf{M}_i^a(\tau) \vec{U}_i^a(x + \tau) d\tau + \mathbf{D}_i^a(\Delta c_i - x) \cdot \mathbf{F}_i^a \hat{\vec{U}}_i(0)$$

where $\vec{U}_i^a(x) = \begin{pmatrix} \vec{U}_i(x) \\ \hline \vec{U}_i'(x) \end{pmatrix} \in \mathbb{R}^{k_i^a \times 1}$, $\vec{U}_i'(x) \in \mathbb{R}^{k_0 \times 1}$ is the vector representing reachability probability for the augmented states in $\mathcal{G}_i$, $\mathbf{F}_i^a = \left(\mathbf{F}_i' \middle| \mathbf{B}_i' \right) \in \mathbb{R}^{k_i^a \times (k_{i+1} + k_0)}$ such that

$\mathbf{F}'_i = \begin{pmatrix} \mathbf{F}_i \\ \hline \mathbf{0} \end{pmatrix} \in \mathbb{R}^{k_i^a \times k_{i+1}}$ is the incidence matrix for delay edges and $\mathbf{B}'_i = \begin{pmatrix} \mathbf{0} \\ \hline \mathbf{I} \end{pmatrix} \in \mathbb{R}^{k_i^a \times k_0}$, $\vec{\hat{U}}_i(0) = \begin{pmatrix} \vec{U}_{i+1}(0) \\ \hline \vec{U}_0(0) \end{pmatrix} \in \mathbb{R}^{(k_{i+1}+k_0)\times 1}$.

First, we prove the following equation:

$$\vec{U}_i^a(x) = \mathbf{\Pi}_i^a(\Delta c_i - x) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0),$$

where

$$\mathbf{\Pi}_i^a(x) = \int_0^x \mathbf{M}_i^a(\tau)\mathbf{\Pi}_i^a(x-\tau)d\tau + \mathbf{D}_i^a(x). \tag{26}$$

We consider the iterations of the solution of the following system of integral equations: set $c_{i,x} = \Delta c_i - x$.

$$\begin{aligned} \vec{U}_i^{a,(0)}(x) &= \vec{0} \\ \vec{U}_i^{a,(j+1)}(x) &= \int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\vec{U}_i^{a,(j)}(x+\tau)d\tau + \mathbf{D}_i^a(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0). \end{aligned}$$

and

$$\begin{aligned} \mathbf{\Pi}_i^{a,(0)}(c_{i,x}) &= \mathbf{0} \\ \mathbf{\Pi}_i^{a,(j+1)}(c_{i,x}) &= \int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\mathbf{\Pi}_i^{a,(j)}(c_{i,x}-\tau)d\tau + \mathbf{D}_i^a(c_{i,x}). \end{aligned}$$

By induction on j, we prove the following relation:

$$\vec{U}_i^{a,(j)}(x) = \mathbf{\Pi}_i^{a,(j)}(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0).$$

- Base case: $\vec{U}_i^{a,(0)}(x) = \vec{0}$ and $\mathbf{\Pi}_i^{a,(0)}(c_{i,x}) = \mathbf{0}$.
- Induction hypothesis: $\vec{U}_i^{a,(j)}(x) = \mathbf{\Pi}_i^{a,(j)}(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0)$.
- Induction step $j \to j+1$:

$$\vec{U}_i^{a,(j+1)}(x) = \int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\vec{U}_i^{a,(j)}(x+\tau)d\tau + \mathbf{D}_i^a(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0).$$

By induction hypothesis we have

$$\begin{aligned} \vec{U}_i^{a,(j+1)}(x) &= \int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\vec{U}_i^{a,(j)}(x+\tau)d\tau + \mathbf{D}_i^a(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0) \\ &= \int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\mathbf{\Pi}_i^{a,(j)}(c_{i,x}-\tau) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0)d\tau + \mathbf{D}_i^a(c_{i,x}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0) \\ &= \left(\int_0^{c_{i,x}} \mathbf{M}_i^a(\tau)\mathbf{\Pi}_i^{a,(j)}(c_{i,x}-\tau)d\tau + \mathbf{D}_i^a(c_{i,x}) \right) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0) \\ &= \mathbf{\Pi}_i^{a,(j+1)}(c_{i,x}) \cdot \mathbf{F}_i \vec{\hat{U}}_i(0). \end{aligned}$$

Clearly, $\mathbf{\Pi}_i^a(c_{i,x}) = \lim_{j\to\infty} \mathbf{\Pi}_i^{a,(j+1)}(c_{i,x})$ and $\vec{U}_i^a(x) = \lim_{j\to\infty} \vec{U}_i^{a,(j+1)}(x)$.

Let $x = 0$ and we obtain

$$\vec{U}_i^a(0) = \mathbf{\Pi}_i^a(c_{i,0}) \cdot \mathbf{F}_i^a \vec{\hat{U}}_i(0).$$

We can also write the above relation for $x = 0$ as:

$$\begin{aligned}
\left(\frac{\vec{U}_i(0)}{\vec{U}'_i(0)}\right) &= \mathbf{\Pi}^a_i(\Delta c_i)\left(\mathbf{F}'_i \middle| \mathbf{B}'_i\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\begin{array}{c|c} \mathbf{\Pi}_i(\Delta c_i) & \bar{\mathbf{\Pi}}^a_i(\Delta c_i) \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\begin{array}{c|c} \mathbf{F}_i & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\begin{array}{c|c} \mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i & \bar{\mathbf{\Pi}}^a_i(\Delta c_i) \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)\left(\frac{\vec{U}_{i+1}(0)}{\vec{U}_0(0)}\right) \\
&= \left(\frac{\mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i\vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}^a_i(\Delta c_i)\vec{U}_0(0)}{\vec{U}_0(0)}\right).
\end{aligned}$$

As a result we can represent $\vec{U}_i(0)$ in the following matrix form

$$\vec{U}_i(0) = \mathbf{\Pi}_i(\Delta c_i)\mathbf{F}_i\vec{U}_{i+1}(0) + \bar{\mathbf{\Pi}}^a_i(\Delta c_i)\vec{U}_0(0)$$

by noting that $\mathbf{\Pi}_i$ is formed by the first k_i rows and columns of matrix $\mathbf{\Pi}^a_i$ and $\bar{\mathbf{\Pi}}^a_i$ is formed by the first k_i rows and the last $k^a_i - k_i$ columns of $\mathbf{\Pi}^a_i$.

For $i = m$, i.e., the last graph $\mathcal{G}_m$, the region size is infinite, therefore delay transitions do not exist. The vector $\vec{U}_m(x+\tau)$ in $\int_0^\infty \hat{\mathbf{M}}_m(\tau)\vec{U}_m(x+\tau)d\tau$ does not depend on entering time x, therefore we can take it out of the integral. As a result we obtain $\int_0^\infty \hat{\mathbf{M}}_m(\tau)d\tau \cdot \vec{U}_m(0)$. More than that $\int_0^\infty \hat{\mathbf{M}}_m(\tau)d\tau$ boils down to $\hat{\mathbf{P}}_m$ and $\int_0^\infty \mathbf{B}_m(\tau)d\tau$ to $\hat{\mathbf{B}}_m$. Also we add the vector $\vec{1}_F$ to ensure that the probability to start from a state in V_F is one (see (9)). ∎

Since the coefficients of the linear equations are all known, solving the system of linear equations yields $\vec{U}_0(0)$, which contains the probability $Prob_{v_0}(0)$ of reaching V_F from initial vertex v_0.

Now we explain how (24) is derived from (17). The term $\mathbf{\Pi}_i(\Delta c_i)\cdot\mathbf{F}_i\vec{U}_{i+1}(0)$ is for the delay transitions, where $\mathbf{F}_i$ specifies how the delay transitions are connected between $\mathcal{G}_i$ and $\mathcal{G}_{i+1}$. The term $\bar{\mathbf{\Pi}}^a_i(\Delta c_i)\cdot\vec{U}_0(0)$ is for Markovian transitions with reset. $\bar{\mathbf{\Pi}}^a_i(\Delta c_i)$ in the augmented CTMC $\mathcal{C}^a_i$ specifies the probabilities to take first transitions inside $\mathcal{G}_i$ and then a one-step Markovian transition back to $\mathcal{G}_0$. Eq. (25) is derived from (18). Since it is the last region and time goes to infinity, the time to enter the region is irrelevant (thus set to 0). Thus $\int_0^\infty \hat{\mathbf{M}}_i(\tau)d\tau$ boils down to $\hat{\mathbf{P}}_i$. In fact, the Markovian jump probability inside $\mathcal{G}_m$ can be taken from the embedded DTMC of $\mathcal{C}_m$, which is $\hat{\mathbf{P}}_i$.

Example 10. For the single-clock DMTA$^\diamond$ in Fig. 4(a) (page 13), we show how to compute the reachability probability $Prob((v_0,0),(v_5,\cdot))$ on the region graph $\mathcal{G}$ (cf. Fig. 4(d)), which has been partitioned into subgraphs $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ as in Fig. 8.

The matrices for $\mathcal{G}_0$ are given as

$$\mathbf{M}_0(x) = \begin{pmatrix} 0 & 1\cdot r_0\cdot e^{-r_0 x} & 0 \\ 0.5\cdot r_1\cdot e^{-r_1 x} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{F}_0 = \begin{pmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \end{pmatrix}$$

The matrices for $\mathcal{G}_1$ are given as

$$\mathbf{M}_1(x) = \begin{pmatrix} 0 & r_0\cdot e^{-r_0 x} & 0 & 0 \\ 0&0&0&0 \\ 0&0&0& r_2\cdot e^{-r_2 x} \\ 0&0&0&0 \end{pmatrix} \quad \mathbf{F}_1 = \begin{pmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{pmatrix} \quad \mathbf{B}_1 = \begin{pmatrix} 0&0&0 \\ 1&0&1 \\ 0&0&0 \\ 0&0&0 \end{pmatrix}$$

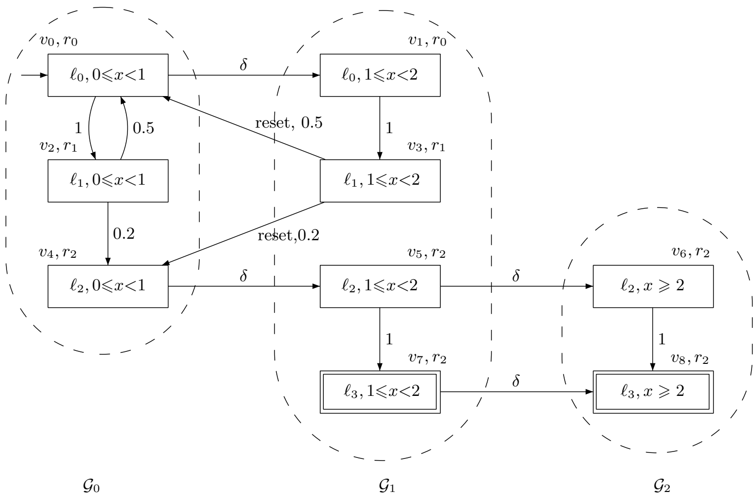


Fig. 8. Partition the region graph in Fig. 4(d)

$$\mathbf{M}_1^a(x) = \begin{pmatrix} 0 & r_0{\cdot}e^{-r_0x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5{\cdot}r_1{\cdot}e^{-r_1x} & 0 & 0.2{\cdot}r_1{\cdot}e^{-r_1x} \\ 0 & 0 & 0 & r_2{\cdot}e^{-r_2x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrices for $\mathcal{G}_2$ are given as

$$\hat{\mathbf{M}}_2(x) = \begin{pmatrix} 0 & r_2{\cdot}e^{-r_2x} \\ 0 & 0 \end{pmatrix} \quad \hat{\mathbf{P}}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

To obtain the system of linear equations, we need:

$$\mathbf{\Pi}_0(1) = \begin{pmatrix} p_{00} & p_{02} & p_{04} \\ p_{20} & p_{22} & p_{24} \\ p_{40} & p_{42} & p_{44} \end{pmatrix} \quad \mathbf{\Pi}_1(1) = \begin{pmatrix} p_{11} & p_{13} & p_{15} & p_{17} \\ p_{31} & p_{33} & p_{35} & p_{37} \\ p_{51} & p_{53} & p_{55} & p_{57} \\ p_{71} & p_{73} & p_{75} & p_{77} \end{pmatrix} \quad \bar{\mathbf{\Pi}}_1^a(1) = \begin{pmatrix} \bar{p}_{10} & \bar{p}_{12} & \bar{p}_{14} \\ \bar{p}_{30} & \bar{p}_{32} & \bar{p}_{32} \\ \bar{p}_{50} & \bar{p}_{52} & \bar{p}_{54} \\ \bar{p}_{70} & \bar{p}_{72} & \bar{p}_{74} \end{pmatrix}$$

All elements in these $\mathbf{\Pi}$-matrices can be computed by the transient probability in the corresponding CTMCs $\mathcal{C}_0$, $\mathcal{C}_1$ and $\mathcal{C}_1^a$ (cf. Fig. 9).

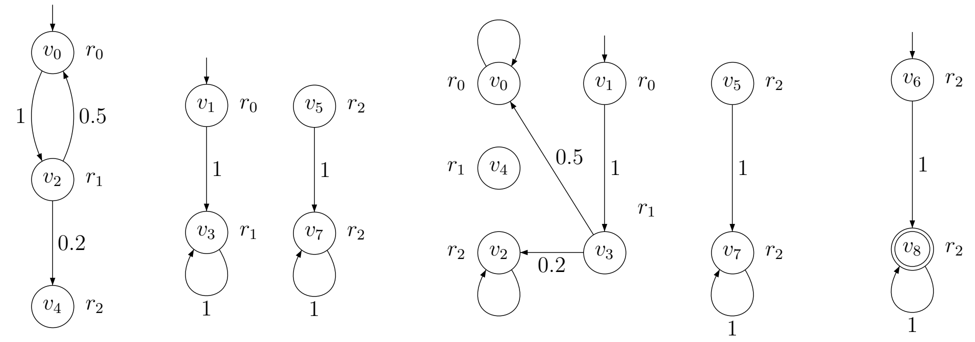
The obtained system of linear equations by applying Theorem 4 is:

$$\begin{pmatrix} u_0 \\ u_2 \\ u_4 \end{pmatrix} = \begin{pmatrix} p_{00} & p_{02} & p_{04} \\ p_{20} & p_{22} & p_{24} \\ p_{40} & p_{42} & p_{44} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_3 \\ u_5 \\ u_7 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ u_3 \\ u_5 \\ u_7 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{13} & p_{15} & p_{17} \\ p_{31} & p_{33} & p_{35} & p_{37} \\ p_{51} & p_{53} & p_{55} & p_{57} \\ p_{71} & p_{73} & p_{75} & p_{77} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_6 \\ u_8 \end{pmatrix} + \begin{pmatrix} \bar{p}_{10} & \bar{p}_{12} & \bar{p}_{14} \\ \bar{p}_{30} & \bar{p}_{32} & \bar{p}_{32} \\ \bar{p}_{50} & \bar{p}_{52} & \bar{p}_{54} \\ \bar{p}_{70} & \bar{p}_{72} & \bar{p}_{74} \end{pmatrix} \cdot \begin{pmatrix} u_0 \\ u_1 \\ u_3 \end{pmatrix}$$

$$\begin{pmatrix} u_6 \\ u_8 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_6 \\ u_8 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

This can be solved easily.



(a) $\mathcal{C}_0$ (b) $\mathcal{C}_1$ (c) $\mathcal{C}_1^a$ (d) $\mathcal{C}_2$

Fig. 9. Derived CTMCs

Remark 6. We note that for two-clock DTA$^\diamond$ which yield two-clock DMTA$^\diamond$, the approach given in this section fails in general. In the single-clock case, the reset guarantees to jump to $\mathcal{G}_0(0)$ and delay to $\mathcal{G}_{i+1}(0)$ when it is in $\mathcal{G}_i$. However, in the two-clock case, after a delay or reset generally only one clock has a fixed value while the value of the other one is not determined.

The time-complexity of computing the reachability probability in the single-clock DTA$^\diamond$ case is $\mathcal{O}(m \cdot |S|^2 \cdot |Loc|^2 \cdot \lambda \cdot \Delta c + m^3 \cdot |S|^3 \cdot |Loc|^3)$, where m is the number of constants appearing in the DTA$^\diamond$, $|S|$ is the number of states in the CTMC, $|Loc|$ is the number of locations in the DTA$^\diamond$, λ is the maximal exit rate in the CTMC and $\Delta c = \max_{0 \leq i < m}\{c_{i+1} - c_i\}$. The first term $m \cdot |S|^2 \cdot |Loc|^2 \cdot \lambda \cdot \Delta c$ is due to the uniformization technique for computing transient distribution; and the second term $m^3 \cdot |S|^3 \cdot |Loc|^3$ is the time complexity for solving a system of linear equations with $\mathcal{O}(m \cdot |S| \cdot |Loc|)$ variables.

5 Model Checking DTA$^\omega$ Specifications

We now deal with DTA$^\omega$ specifications. Given the product $\mathcal{M}^\omega = (Loc, \mathcal{X}, \ell_0, Loc_{\mathcal{F}}, E, \rightsquigarrow)$, we first define the region graph $\mathcal{G}^\omega(\mathcal{M}^\omega)$ (or simply $\mathcal{G}^\omega$) as $(V, v_0, V_F^\omega, \Lambda, \hookrightarrow)$ without specifying how the accepting set V_F^ω is defined. This will become clear later. The elements V, v_0, Λ and $\hookrightarrow$ are defined in the same way as in Def. 10 (page 16).

The Muller acceptance conditions $Q_{\mathcal{F}}$ in the DTA$^\omega$ consider the infinite paths that visit the locations in $F \in Q_{\mathcal{F}}$ infinitely often. For this sake, BSCCs in the *region graph* $\mathcal{G}^\omega$ that consist of set of vertices corresponding to $L_F \in Loc_{\mathcal{F}}$ are of most importance. Note that it is not sufficient to consider the BSCCs in the DMTA$^\omega$. The reason will become clear in Remark 7. Let $v \in B$ denote that vertex v is in the BSCC B. We define *accepting* BSCC*s* as follows:

Definition 12 (aBSCC). *Given a product $\mathcal{C} \otimes \mathcal{A}^\omega = (Loc, \mathcal{X}, \ell_0, Loc_{\mathcal{F}}, E, \rightsquigarrow)$ and its region graph $\mathcal{G}^\omega$, a* BSCC *B in $\mathcal{G}^\omega$ is* accepting *if there exists $L_F \in Loc_{\mathcal{F}}$ such that for any $v \in B$, $v{\downharpoonright_1} \in L_F$. Let $a\mathcal{B}$ denote the set of accepting* BSCC*s in $\mathcal{G}^\omega$.*

Based on $a\mathcal{B}$, we can now define the set of accepting vertices of $\mathcal{G}^\omega$ as $V_F^\omega = \{v \in B \mid B \in a\mathcal{B}\}$. Note that it is *not* an acceptance family but a set of accepting vertices.

Example 11. For the DMTA$^\omega$ in Fig. 5(c) with $Loc_{\mathcal{F}} = \big\{\{\ell_1, \ell_2, \ell_3\}, \{\ell_4, \ell_5, \ell_6\}\big\}$, the region graph is as in Fig. 10. There is one accepting BSCC, which has been labeled with gray. This BSCC corresponds to the set $\{\ell_4, \ell_5, \ell_6\} \in Loc_{\mathcal{F}}$ in the DMTA$^\omega$. There is no BSCC corresponding to the set $\{\ell_1, \ell_2, \ell_3\}$ because in the region graph v_{12} and v_{14} are sink vertices connecting to the SCC. In other words, the probabilities will leak when $x > 2$ on either ℓ_1 or ℓ_2. This is determined by the guards on the DTA$^\omega$.

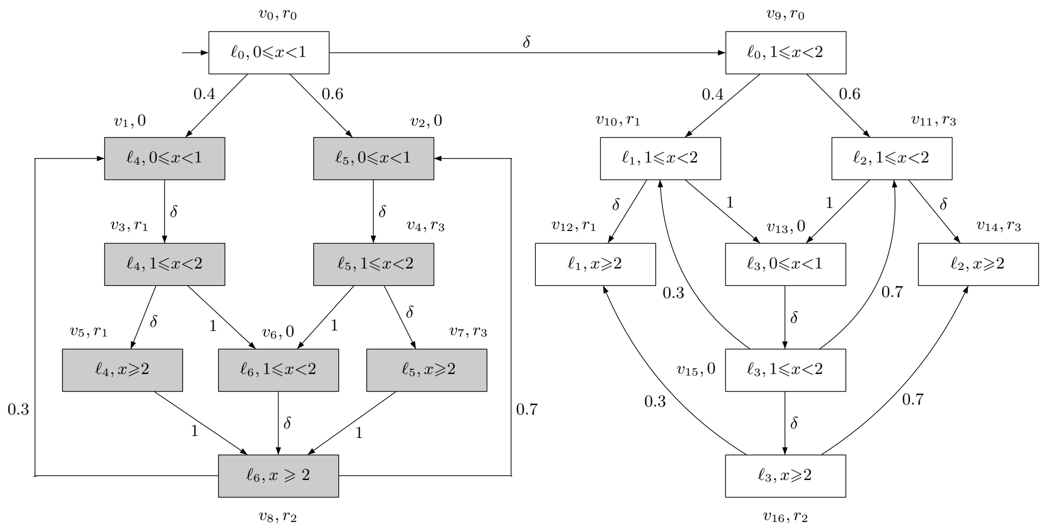


Fig. 10. Region graph of the product DMTA$^\omega$ in Fig. 5(c)

We remark on two points: 1) the probability of staying in an aBSCC is 1, considering both the delay and Markovian transitions. That is to say, there are no outgoing transitions from which probabilities can "leak"; 2) any two aBSCCs are disjoint, such that the probabilities to reach two BSCCs can be added. These two points are later important for the computation of the reachability probability.

Let $Prob^{\mathcal{C}}(\mathcal{A}^\omega)$ be the probability of the set of infinite paths in $\mathcal{C}$ that can be accepted by $\mathcal{A}^\omega$. The following theorem computes $Prob^{\mathcal{C}}(\mathcal{A}^\omega)$ on the region graph:

Theorem 5. *For any* CTMC $\mathcal{C}$*,* DTA$^\omega$ $\mathcal{A}^\omega$*, and the region graph* $\mathcal{G}^\omega = (V, v_0, V_F^\omega, \Lambda, \hookrightarrow)$ *of the product, it holds that:*

$$Prob^{\mathcal{C}}(\mathcal{A}^\omega) = Prob^{\mathcal{G}^\omega}(v_0, \Diamond V_F^\omega).$$

Proof. We show the theorem by the following three steps:

1. $Prob^{\mathcal{C}}(\mathcal{A}^\omega) = Prob^{\mathcal{C}\otimes\mathcal{A}}(Loc_{\mathcal{F}})$, where $Prob^{\mathcal{C}\otimes\mathcal{A}}(Loc_{\mathcal{F}})$ denotes the probability of accepting paths of DMTA $\mathcal{C} \otimes \mathcal{A}$ w.r.t. Muller accepting conditions;
2. $Prob^{\mathcal{C}\otimes\mathcal{A}}(Loc_{\mathcal{F}}) = Prob^{\mathcal{G}^\omega}(v_0, Loc_{\mathcal{F}})$;
3. $Prob^{\mathcal{G}^\omega}(v_0, Loc_{\mathcal{F}}) = Prob^{\mathcal{G}^\omega}(v_0, \Diamond\, V_F^\omega)$.

For the first step, we note that $Paths^{\mathcal{C}}(\mathcal{A}^\omega) = \bigcap_{1\leqslant i\leqslant k} Paths^i$ where

$$Paths^i = \bigcap_{n\geqslant 0} \bigcup_{m\geqslant n} \bigcup_{s_0,\ldots,s_n,s_{n+1}\cdots,s_m} C(s_0, I_0, \ldots, I_{n-1}, s_n, \ldots, I_{m-1}, s_m), \qquad \text{where}$$

– $\{s_{n+1}, \ldots, s_m\} = L_{F_i}$;
– $C(s_0, I_0, \ldots, I_{n-1}, s_n, \ldots, I_{m-1}, s_m)$ is the cylinder set such that each timed path of the cylinder set of the form $s_0 \xrightarrow{t_0} \cdots \xrightarrow{t_{n-1}} s_n \cdots \xrightarrow{t_{m-1}} s_m$ is a prefix of an accepting path of $\mathcal{A}$.

Similar to Lemma 1, one can easily see that each path of CTMC $\mathcal{C}$ can be lifted to a unique path of DMTA$^\omega$ $\mathcal{C} \otimes \mathcal{A}^\omega$. Following the same argument as in Theorem 2, one can obtain that for each cylinder set of the form $C(s_0, I_0, \ldots, I_{n-1}, s_n, \ldots, I_{m-1}, s_m)$, $\mathcal{C}$ and $\mathcal{C} \otimes \mathcal{A}^\omega$ give rise to the same probability. Hence $Prob^{\mathcal{C}}(\mathcal{A}^\omega) = Prob^{\mathcal{C}\otimes\mathcal{A}^\omega}(Loc_{\mathcal{F}})$.

For the second step, we need to define a timed path of $\mathcal{G}^\omega$, which is of the form $v_0 \xrightarrow{t_0} v_1 \xrightarrow{t_2} \cdots$ such that given the initial valuation η_0, one can construct a sequence $\{\eta_i\}$ such that

- $\eta_{i+1} = (\eta_i + t_i)[X_i := 0]$ if $\eta_i + t_i \models Inv(v_i)$ (namely, the transition from v_i to v_{i+1} is via a Markovian transition); and
- $\eta_{i+1} = \eta_i + t_i$ if $\eta_i + t_i \in \partial Inv(v_i)$ (namely, the transition from v_i to v_{i+1} is via a forced boundary jump).

A path of $\mathcal{G}^\omega$ is accepted if the discrete part of the path, namely $v_0 v_1 \cdots$ meets the Muller condition.

Following the standard region construction, one can lift a timed path of DMTA$^\omega$ $\mathcal{C}\otimes\mathcal{A}^\omega$ to a unique timed path of the corresponding region graph $\mathcal{G}^\omega$. Moreover, following the same argument of Theorem 3, one can show that $\mathcal{C} \otimes \mathcal{A}^\omega$ and $\mathcal{G}^\omega$ give rise to the same probability to the accepted paths.

For the third step, we note that according to the ergodicity of PDP (region graph), for each path of $\mathcal{G}^\omega$, with probability 1 the states visited infinitely often constitute a BSCC. It follows that

$$Prob^{\mathcal{G}^\omega}(v_0, Loc_{\mathcal{F}}) = \sum_{B \in a\mathcal{B}} Prob\{\rho \mid inf(\rho) = B\}.$$

We note that for each note v in an accepting BSCC, $Prob\{Paths^{\mathcal{G}^\omega}(v)\} = 1$. Hence

$$Prob^{\mathcal{G}^\omega}(v_0, Loc_{\mathcal{F}}) = Prob^{\mathcal{G}^\omega}(v_0, \Diamond V_F^\omega).$$

■

Actually, the region graph $\mathcal{G}^\omega$ can be simplified to $\mathcal{G}^\omega_{abs}$ to compute $Prob^{\mathcal{C}}(\mathcal{A}^\omega)$. $\mathcal{G}^\omega_{abs}$ is obtained by making (i) all vertices in V_F^ω and (ii) all vertices that *cannot* reach V_F^ω absorbing. (i) is justified by the fact that for these $v \in V_F^\omega$, $Prob^{\mathcal{G}}(v, \Diamond V_F^\omega) = 1$; while (ii) is because $Prob^{\mathcal{G}}(v', \Diamond V_F^\omega) = 0$, for v' cannot reach V_F^ω. It is obvious to see that

$$Prob^{\mathcal{G}^\omega}(v_0, \Diamond V_F^\omega) = Prob^{\mathcal{G}^\omega_{abs}}(v_0, \Diamond V_F^\omega).$$

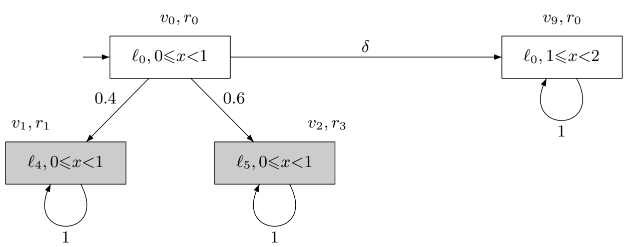


Fig. 11. The transformed region graph $\mathcal{G}^\omega_{abs}$

Example 12. The transformed region graph $\mathcal{G}^\omega_{abs}$ of that in Fig. 10 is shown in Fig. 11. We omit all the vertices that cannot be reached from v_0 in $\mathcal{G}^\omega_{abs}$. In this new model, $V_F^\omega = \{v_1, v_2\}$. We now can perform the approach for computing timed-unbounded reachability probabilities in Section 4 such that Eq. (7)-(9) can be applied. We have:

$Prob^{\mathcal{G}^{\omega}_{abs}}(v_0, \Diamond V^{\omega}_F) = Prob^{\mathcal{G}^{\omega}_{abs}}(v_0, \Diamond at_{v_1}) + Prob^{\mathcal{G}^{\omega}_{abs}}(v_0, \Diamond at_{v_2})$. Note that $Prob^{\mathcal{G}^{\omega}_{abs}}(v_i, \Diamond V^{\omega}_F) = 1$ for $i = 1, 2$ and 0 for $i = 9$. For the delay transition $v_0 \overset{\delta}{\hookrightarrow} v_9$,

$$Prob_{v_0,\delta}(0) = e^{-r_0 \cdot 1} \cdot Prob_{v_9}(1) = e^{-r_0 \cdot 1} \cdot 0 = 0.$$

For the Markovian transition $v_0 \overset{0.4,\{x\}}{\hookrightarrow} v_1$,

$$Prob_{v_0,v_1}(0) = \int_0^1 0.4 \cdot r_0 \cdot e^{-r_0 \cdot \tau} \cdot Prob_{v_1}(\tau) d\tau = \int_0^1 0.4 \cdot r_0 \cdot e^{-r_0 \cdot \tau} d\tau.$$

A similar reasoning applies to $v_0 \overset{0.6,\{x\}}{\hookrightarrow} v_2$. In the end, we have

$$Prob^{\mathcal{C}}(\mathcal{A}^{\omega}) = \int_0^1 (0.4 + 0.6) \cdot r_0 \cdot e^{-r_0 \cdot \tau} d\tau = \int_0^1 r_0 \cdot e^{-r_0 \cdot \tau} d\tau = 1 - e^{-r_0}.$$

Remark 7 (Why not BSCC*s in the product?).* There are two BSCCs in the product DMTA$^{\omega}$: one formed by $\{\ell_1, \ell_2, \ell_3\}$ and the other by $\{\ell_4, \ell_5, \ell_6\}$. As turned out in the example that only the latter forms a BSCC in the region graph while the former does not. This is because the guards on the transitions also play a role on whether a path can be accepted. The impact of guards, however, is not immediately clear in the product DMTA$^{\omega}$, but is implicitly consumed in the region graph. This justifies finding BSCCs in the region graph instead of in the product.

Theorem 5 implies that computing the probability of a set of infinite paths (LHS) can be reduced to computing the probability of a set of finite paths (RHS). The latter has been solved in Section 4 with the characterization of a system of integral equations and also the approximation by a system of PDEs. The case of a single clock DTA$^{\omega}$, due to this reduction, can also be solved as a system of ODEs (as in Section 4.2).

6 Conclusion

We addressed the quantitative verification of a CTMC $\mathcal{C}$ against a DTA$^{\Diamond}$ $\mathcal{A}$ (DTA$^{\omega}$ $\mathcal{A}^{\omega}$). As a key result, we showed that the set of the accepting paths in $\mathcal{C}$ by DTA is measurable and the probability of $\mathcal{C} \models \mathcal{A}$ can be reduced to computing reachability probabilities in the embedded DTMP of a PDP. The probabilities can be characterized by a system of Volterra integral equations of the second type and can be approximated by a system of PDEs. For single-clock DTA$^{\Diamond}$, this reduces to solving a system of linear equations whose coefficients are a system of ODEs. The probability of $\mathcal{C} \models \mathcal{A}^{\omega}$ is reducible to computing the reachability probabilities to the accepting BSCCs in the region graph and the thus obtained PDP.

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